Earthquake excitation

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Earthquake
excitation

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Earthquakes and
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Outline

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Seismic Excitation

The most important quantity related to earthquake excitation is the ground acceleration.

Ground acceleration can be recorded with an accelerometer, basically a SDOF oscillator, with a damping ratio $\zeta \approx 70\%$, whose displacements are proportional to ground accelerations up to a given frequency.

Instrument records of *strong ground motion* first became available in the '30s, the first record of a destructive ground motion being the 1940 records of El Centro earthquake.

Until recently, we have tons of records for a few earthquakes that happened in California and Japan, recorded in many different sites and locations (in the free field, on building foundations, on different building storeys etc), less so for different areas. Now, in many countries building codes requires that important constructions must be equipped with accelerometers and many national agencies own and operate strong motion networks, so that the availability of strong motion records is constantly improving.
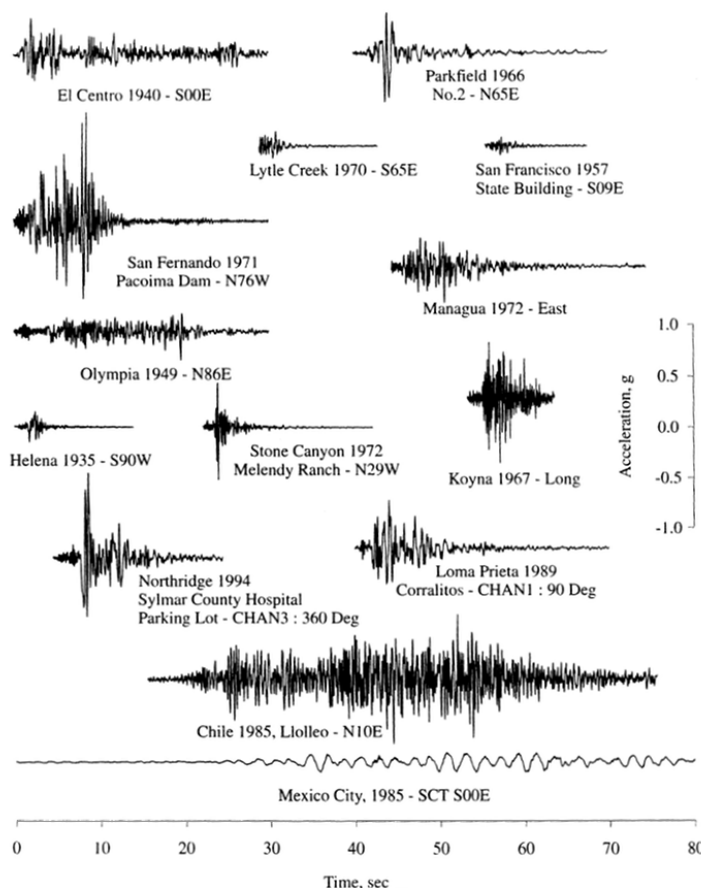
Seismic Excitation Samples

In the figure on the left, a number of different ground acceleration time histories, recorded at different sites and due to different earthquakes, are plotted with the same scale, both in time and in acceleration.

You should appreciate the large variability in terms of amplitudes, duration, frequency content and general appearance of different earthquake records.

Detailed Sample
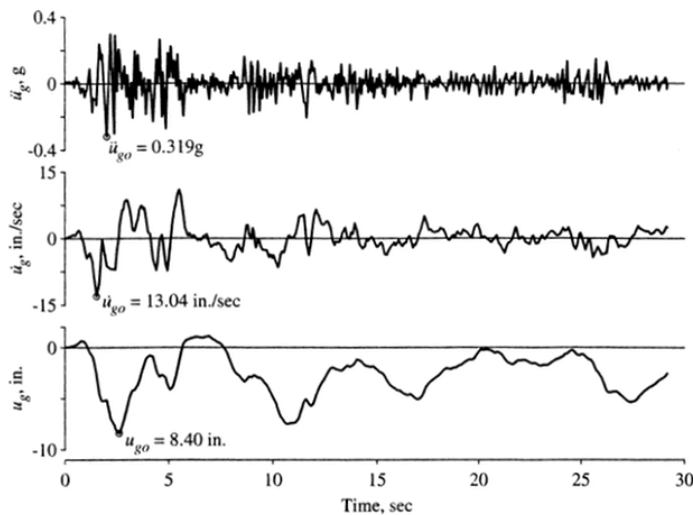
Figure 6.1.4 North–south component of horizontal ground acceleration recorded at the Imperial Valley Irrigation District substation, El Centro, California, during the Imperial Valley earthquake of May 18, 1940. The ground velocity and ground displacement were computed by integrating the ground acceleration.

Above, the acceleration recorded at El Centro during the Imperial Valley 1940 e.q., along with the velocity and displacements obtained by numerical integration. For meaningful computations, it is very consequential the choice of initial conditions and the ability to remove any possible trend in the acceleration record.

About Ground Motion

Tough accelerations vary with time in a very irregular manner, the variation is fully known and is assumed to be independent from the presence of a building.

To describe such an irregular motion, an high number of samples is required, modern instruments record the acceleration at a rate of 200 and more samples per second.

The equation of motion is

$$\ddot{D} + 2\zeta\omega\dot{D} + \omega^2 D = -\ddot{u}_g(t),$$

due to the irregular nature of ground excitation the response must be evaluated numerically.

Clearly, the displacement response function, for assigned $\ddot{u}_g$, depends on ζ and ω only.
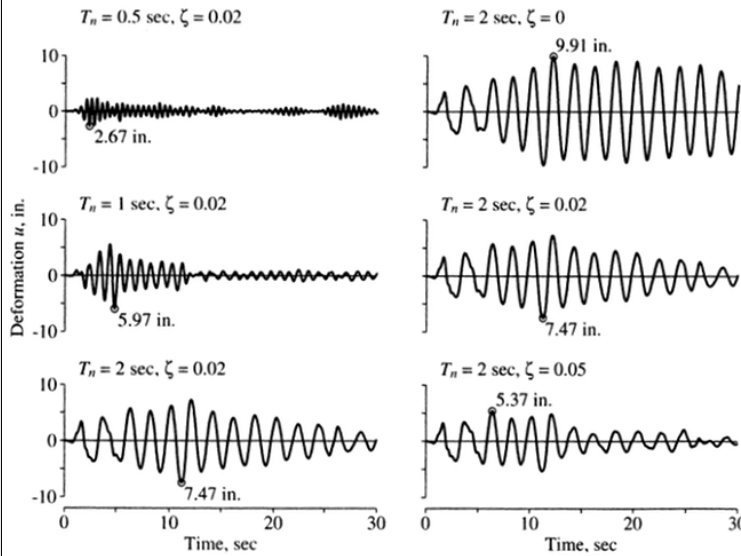
T_n and ζ dependency

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Displacement response functions for the El Centro 1940 NS acceleration record.

Leftmost column, fixed $\zeta = 0.02$ and varying T_n (in earthquake engineering it is more usual to use the natural period than the circular frequency), $T_N = 0.5, 1.0, 2.0\,\text{s}$, note that, although the ground motion is irregular, the responses are similar, each one having a period close to T_n.

Centre column, $T_n = 2.0\,\text{s}$ and varying $\zeta = 0, 0.02, 0.05$, note that for a fixed period the shape of responses is similar and that the maximum response value depends on ζ.

Pseudo Acceleration

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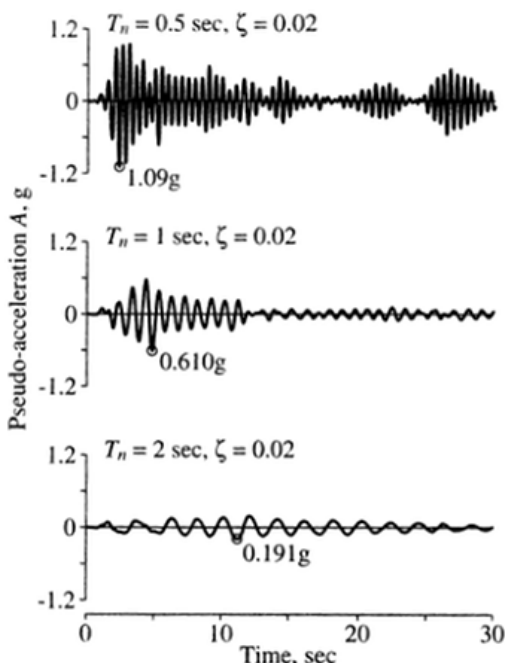
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From deformation response, we can compute the equivalent static force

$$f_s(t) = m\omega^2 D(t) = m A(t)$$

where $A(t)$ is the pseudo acceleration, $A(t) = \omega_n^2 D(t) = (2\pi)^2 D(t)/T_n^2$
Note, ome more time, that f_s is proportional to $A(t)$ and not to the acceleration $\ddot{D}(t)$.

Left, pseudo accelerations computed for varying T_n. Compare with previous page's figure. The relative magnitudes are reverted: for $T_n = 0.5\,\text{s}$ we have a maximum force and a minimum displacement, while for $T_n = 2.0\,\text{s}$ the force is minimum and the displacement maximum.

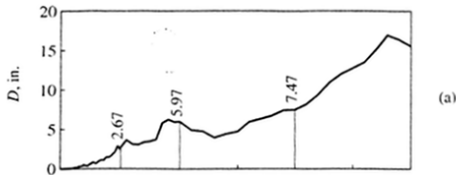# Response Spectrum

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Response
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Combined
$D - V - A$
spectrum
Tripartite Plots
Example of Use of
the Spectrum

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Introduced by M.A. Biot in 1932, popularised by G.W. Housner, the concept of response spectrum is instrumental to characterise e.q. response.

The response spectrum is a plot of the peak values of a response quantity, say the displacement response function, computed for different values of T_n and the same ζ, versus natural period T_N.

A graph where several such plots, obtained for different values of ζ, representative of different damping ratios that characterise different structures, are plotted close to each other represents the e.q. characteristics from the point of view of peak structural response (wait later slides for examples).

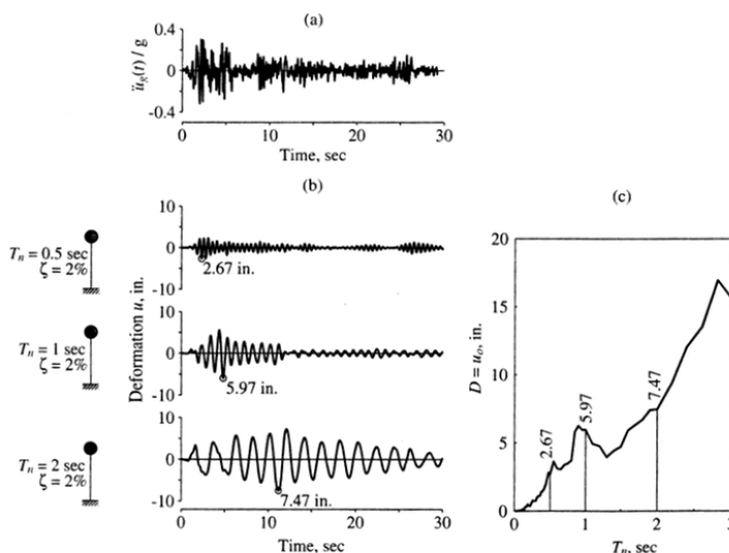# Computing the DRS

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For fixed values of ζ (usually chosen within 0% 0.5% 1% 2% 3% 5% 7% 10% 15% and 20%) and for variable values of T_n (usually ranging from 0.01 s to 20 s) the displacement response function is numerically integrated, the peak value is individuated, the peak value is plotted.

Pseudo Spectra

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Only the Deformation Response Spectrum (DRS) is required to fully characterise the peaks of deformations and equivalent static forces.
It is however useful to study also the pseudo acceleration (PARS) and pseudo velocity (PVRS) spectra, as they are useful in understanding excitation intrinsic characteristics, in constructing *design spectra* and to connect dynamics and building codes.
We have already introduced $A(t)$, consider now the quantity

$$V(t) = \omega_n D(t) = \frac{2\pi}{T_n} D(t)$$

that is, the pseudo velocity.
The peak value of V is connected with the maximum strain energy,

$$E_{s,0} = \frac{1}{2} m V_0$$

being $E_{s,0} = \frac{1}{2} D_0 \, m\omega^2 D_0$. Once again, $V \neq \dot{x}$, the relative velocity.
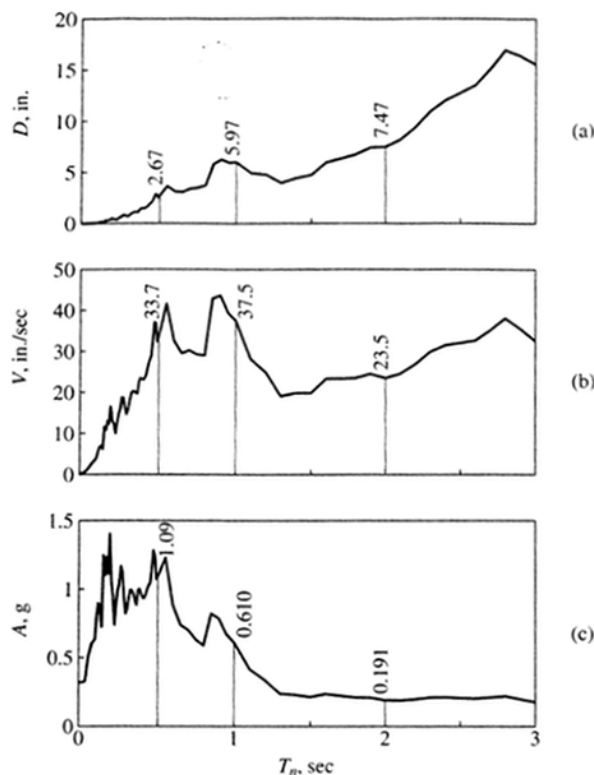

Pseudo Spectra

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Deformation spectrum, pseudo velocity and pseudo acceleration spectra
for El Centro 1940 NS, $\zeta = 2\%$.

Combined $D - V - A$ spectrum

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In the following, we will use the symbols D, V and A to represent the values of the DRS, PVRS and PARS spectra, respectively, with

$$V = \omega_n D, \qquad A = \omega_n^2 D$$

While D, V and A represent the same information, nonetheless it is useful to maintain a distinction as they are connected to different response quantities, the maximum deformation, the maximum strain energy and the maximum equivalent static force.

Moreover, it is possible to plot all three spectra on the same logarithmic plot, giving what is regarded as a fundamental insight into the ground motion characteristics.

Constant A

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Consider a plane with axes $\log T_n$ and $\log V$, and the locus of this plane where A is constant, $A = \hat{A}$:
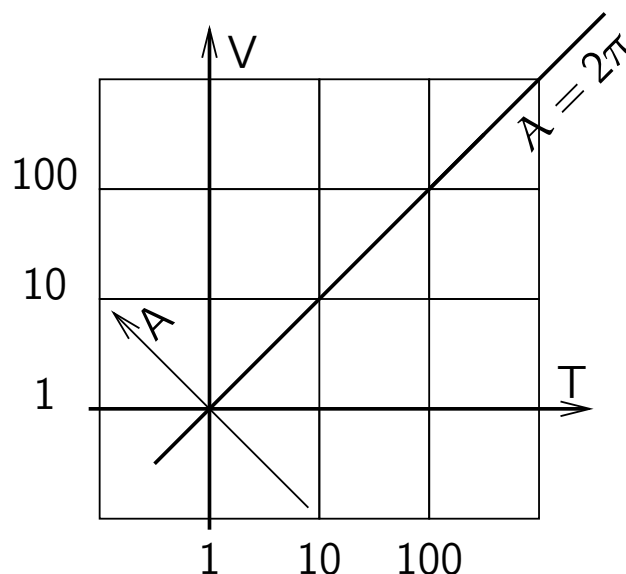
it is

$$A = 2\pi V / T_n = \hat{A}$$

taking the logarithm

$$\log \frac{\hat{A}}{2\pi} = \log V - \log T_n$$

or

$$\log V = \log T_n + \log \frac{\hat{A}}{2\pi}.$$

In the log-log plane straight lines at $45°$ are characterised by a constant value of A.

Constant D

Now, in the same a plane with axes $\log T_n$ and $\log V$ we seek the locus where D is constant, $D = \hat{D}$:
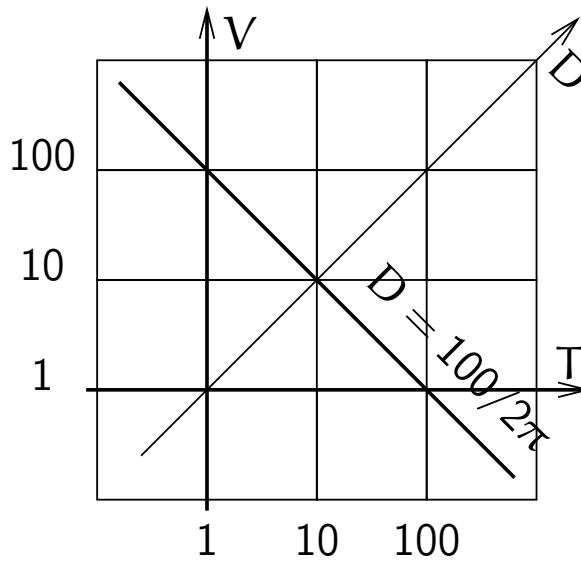
it is

$$D = T_n \, V \, / 2\pi = \hat{D}$$

taking the logarithm

$$\log 2\pi \, \hat{D} = \log V + \log T_n$$

or

$$\log V = \log 2\pi \, \hat{D} - \log T_n.$$

In the log-log plane straight lines at $-45°$ are characterised by a constant value of D.

Example of Construction, 1

Example of Construction, 2

Earthquake
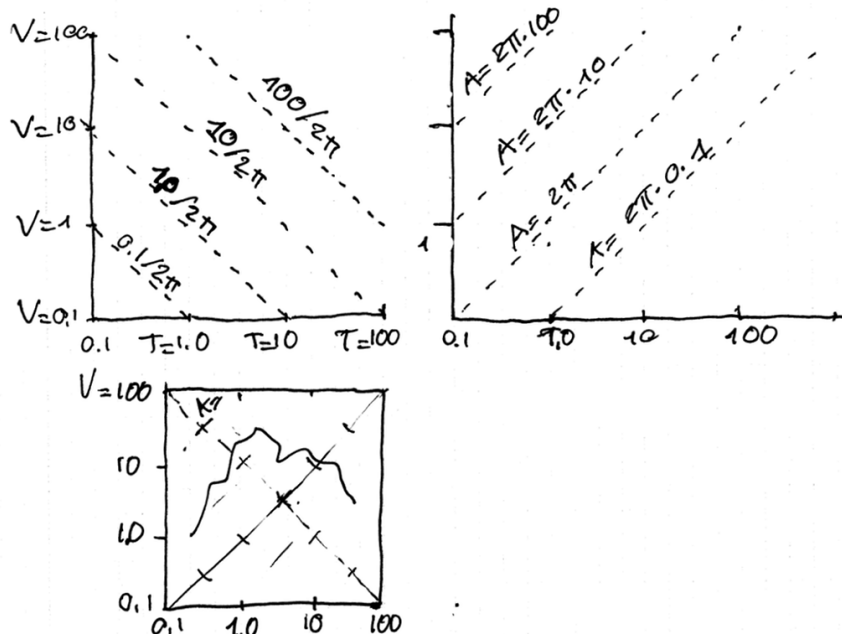excitation

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Response
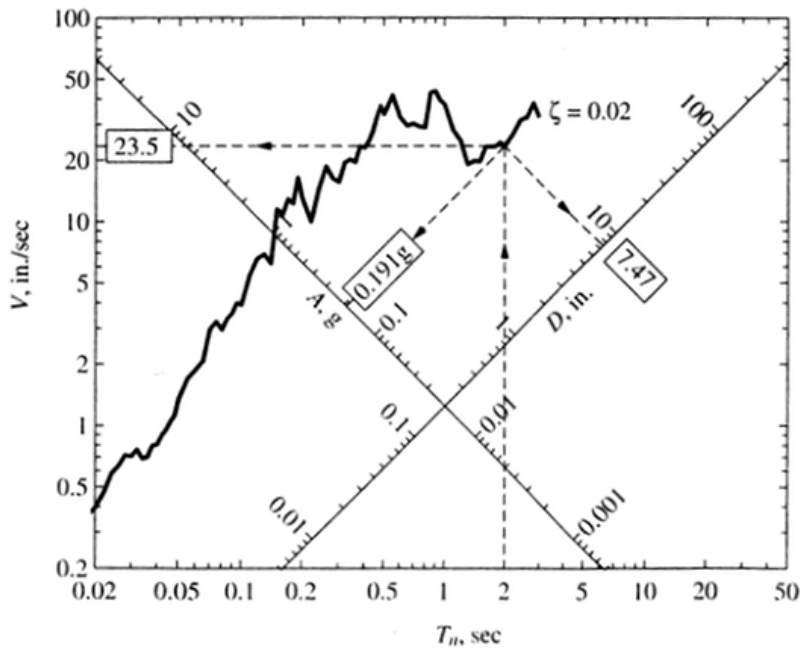Spectrum
**Combined
D − V − A
spectrum**
Tripartite Plots
**Example of Use of
the Spectrum**

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Combined $D − V − A$ response spectrum, El Centro 1940 NS record, $\zeta = 0.02$.

Example of $D − V − A$ spectrum

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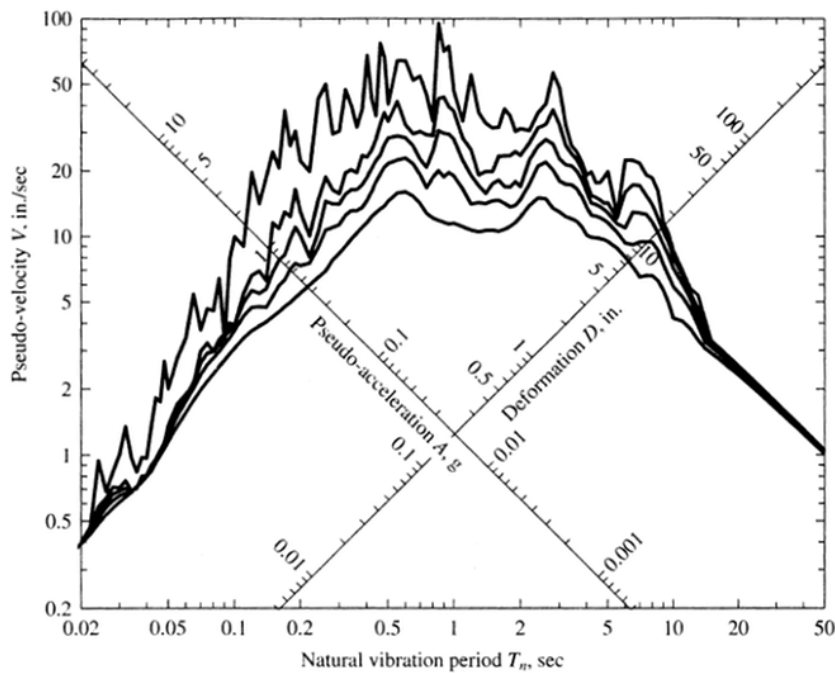
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Combined $D − V − A$ response spectrum, El Centro 1940 NS record,
for $0 \leqslant \zeta \leqslant 20\%$ and full range of periods.

Peak Structural Response

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**Combined
$D - V - A$
spectrum**

Tripartite Plots

**Example of Use of
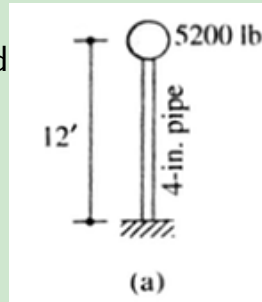the Spectrum**

Response
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The peak deformation u_0 is given by

$$u_0 = D$$

and the peak of the equivalent static force $f_{S,0}$ is given by

$$f_{S,0} = ku_0 = m\omega^2 u_0 = kD = mA$$

It is required to know the peak of the base bending moment for the structure on the right, when subjected to the NS component of the El Centro 1940 record. The mass is $m = 2360\,\text{kg}$, the stiffness is $k = 36.84\,\text{kN}\,\text{m}^{-1}$, the natural period of vibration is computed as $T_n = 1.59\,\text{s}$. The damping ratio is assumed to be $\zeta = 5\%$.

On the graph of the relevant $D - V - A$ spectrum, for $T_n = 1.59$, we find the value $A = 0.20\,\text{g}$.

The equivalent static force is $f_{S,0} = 2360\,\text{kg} \cdot 0.20 \cdot 9.81\,\text{m}\,\text{s}^{-2} = 4.63\,\text{kN}$ and the peak base bending moment is $M_{b,0} = 4.63\,\text{kN} \cdot 12 \cdot 0.305\,\text{m} = 16.93\,\text{kN}\,\text{m}$

5200 lb

12′

4-in. pipe

(a)

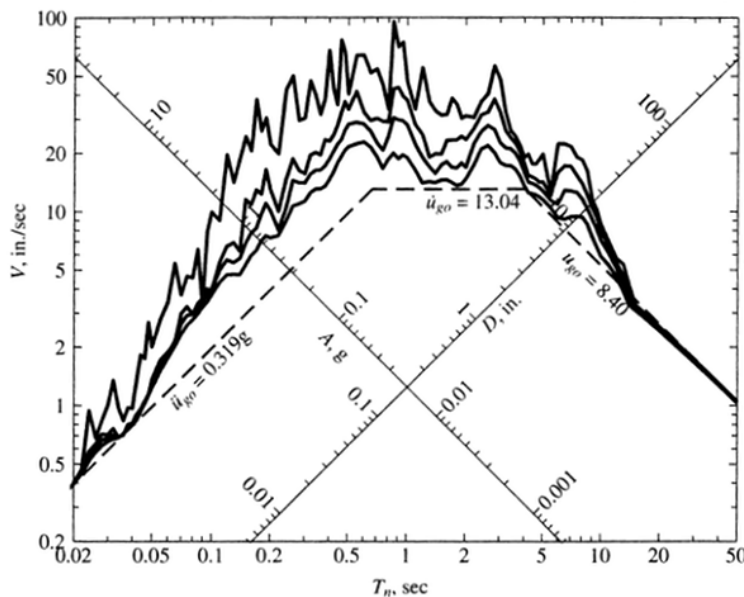# Response Spectrum Characteristics

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**Idealised
Response Spectra**

**Elastic Design
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**Example and
Summary**
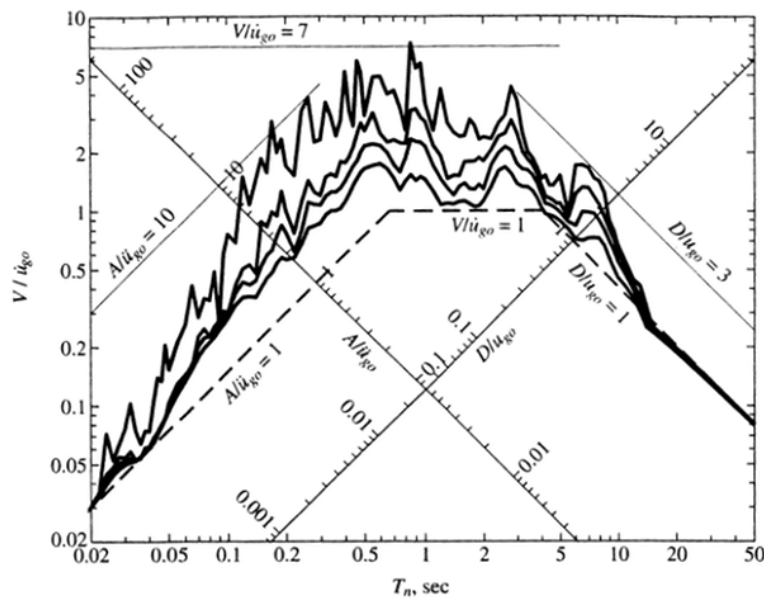
For intermediate values of T_n it is apparent that

- $A > \ddot{u}_{g,0}$, $V > \dot{u}_{g,0}$ and $D > u_{g,0}$;
- $V_{\text{max}} \approx$ constant for each value of ζ,
- there is a clear dependency on ζ.

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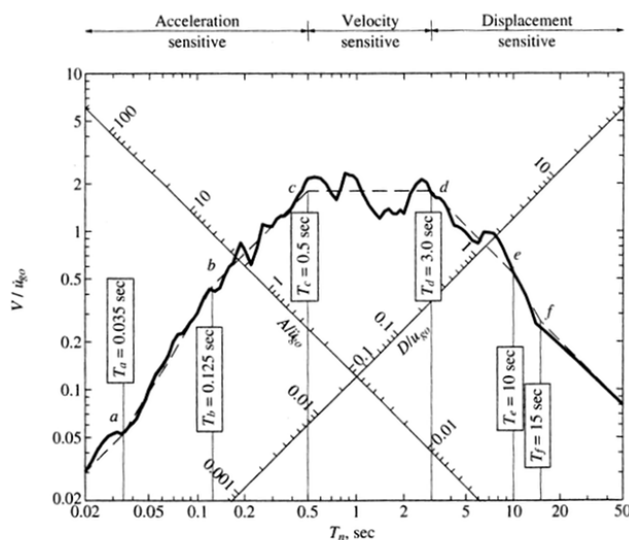Characteristics

**Idealised
Response Spectra**
**Elastic Design
Spectra**
**Example and
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First step in the construction of an idealised $D - V - A$ response spectrum is to make a tripartite plot with all three ordinate axes normalised with respect to $u_{g,0}$, $\dot{u}_{g,0}$ and $\ddot{u}_{g,0}$.

Next,

- ▶ draw the $\zeta = 5\%$ spectrum,
- ▶ individuate the intervals where a) $A \approx \ddot{u}_{g,0}$, b) $A \approx a_A \ddot{u}_{g,0}$, c) $V \approx a_V \dot{u}_{g,0}$, d) $D \approx a_D u_{g,0}$, e) $D \approx u_{g,0}$ and
- ▶ individuate approximate amplifications factors, a_A, a_V and a_D,
- ▶ connect the constant value intervals with straight lines.

Idealised Response Spectrum

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**Idealised
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Our procedure results look good in the log-log graph, but should we represent the same piecewise linearisation in a lin-lin graph it will be apparent that's a rather crude approximation.

This consideration is however not particularly important, because we are not going to use the idealised spectrum in itself, but as a guide to help developing design spectra.

Finally, consider that the positions of the points $T_a, \ldots, T_f$ and the amplifications factors α_A, α_V and α_D are not equal for spectra of different earthquakes recorded at different sites, they depend in complex and not fully determined ways on different parameters, for example the focal distance and the focal mechanism and, very important, the local soil characteristics, showing in the whole a large variability.

Elastic Design Spectra

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Response
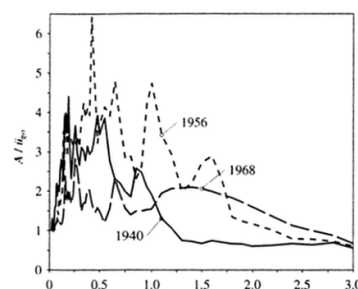Spectrum
Characteristics

Idealised
Response Spectra

**Elastic Design
Spectra**

Example and
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On the right, the $\ddot{u}_{g,0}$ normalised A response spectra for 3 different earthquakes NS records, recorded at the same El Centro site.

Clearly, it is not possible to infer the jagged appearance of the 1968 spectra from the 1940's and 1956's ones.

For design purposes, however, it is not necessary to know in advance and in detail the next quake's response spectra as it suffices to know some sort of an upper bound on spectral ordinates.

Elastic Design Spectra

A design spectrum has not, in contrast with a response spectrum, a jagged appearance because it is specified basing on the concept of idealized response spectrum, as a set of connected straight lines on the log-log $D - V - A$ plot (that maps on a set of straight and curved lines on conventional $T - n — A$ plots).

The requirements of a design spectrum are manifold, but mostly important a design spectrum should be an envelope of possible peak values.

Elastic Design Spectra

The procedure used for computing an elastic design spectrum could be sketched as follows,

- ▶ collect earthquake records from the site under study or from similar sites (similar in local geology, in epicentral distances, duration of strong motion etc) and compute normalized response spectra,

- ▶ statistically characterise, in terms of mean values and standard deviations, the set of normalised spectral ordinates at hand,

- ▶ derive idealized spectra.

Derivation of an Elastic Design Spectra

Riddel and Newmark (1979)
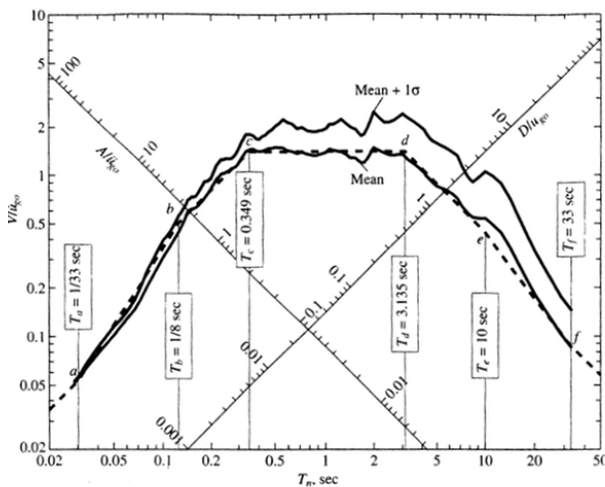
Riddel and Newmark

a collected a large set of records for similar sites in Southern California,

b computed the normalised response spectra for $z = 5\%$ and, finally

c computed the mean value and the standard deviation of the peak response distribution.

In the graph, the summary of their research: the mean and mean+1σ spectra for 5% damping ratio.
In the same graph, you can see also (dashed) an idealised spectrum representation of the mean spectrum.

Idealised Elastic Design Spectra

It is common practice to subdivide the design $D - V - A$ elastic spectrum in 7 segments and use 4 key vibration periods, together with given amplification factors, to draw the required idealised design spectrum.
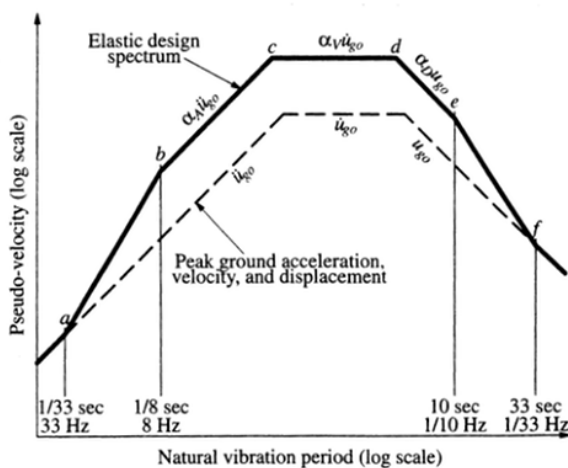
The key periods $T_a = 0.03\,\text{s}$ and $T_b = 0.125\,\text{s}$ define the segment where A rises from 1 to α_A
The key periods $T_e = 10\,\text{s}$ and $T_f = 33\,\text{s}$ define the segment where D decreases from α_D to 1.
The key periods T_c and T_d, instead, follows from applying the given amplification factors to pseudo accelerations, pseudo velocities and deformation.

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Example Data

ζ (%)	Median (50th percentile)			Median+1σ (84th percentile)		
	α_A	α_V	α_D	α_A	α_V	α_D
1	3.21	2.31	1.82	4.38	3.38	2.73
2	2.74	2.03	1.63	3.66	2.92	2.42
5	2.12	1.65	1.39	2.71	2.30	2.01
10	1.64	1.37	1.20	1.99	1.84	1.69
20	1.17	1.08	1.01	1.26	1.37	1.38

	Median	Median+1σ
α_A	$3.21 - 0.68 \log \zeta$	$4.38 - 1.04 \log \zeta$
α_V	$2.31 - 0.41 \log \zeta$	$3.38 - 0.67 \log \zeta$
α_D	$1.82 - 0.27 \log \zeta$	$2.73 - 0.45 \log \zeta$

Source: N.M. Newmark and W.J. Hall, *Earthquake Spectra and Design*, EERC Report 1982.

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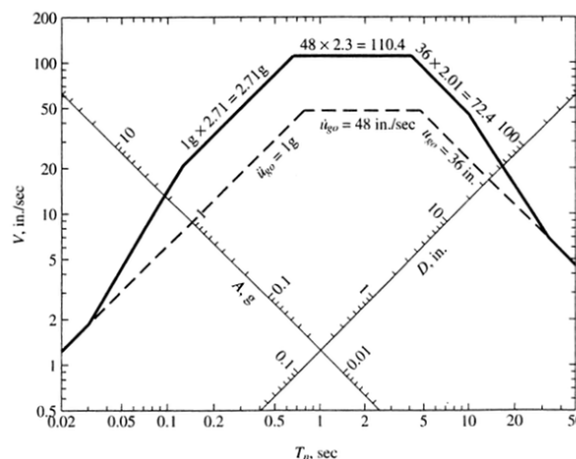Earthquakes and
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Procedure Summary

1. for the site in case, get an estimate of $\ddot{u}_{g,0}$, $\dot{u}_{g,0}$ and $u_{g,0}$, from an analysis of relevant data or desuming it from literature,
2. in the tripartite log-log graph, draw a line for each of the shaking parameters,
3. for a selected value of ζ amplify the shaking parameters by appropriate amplification factor and draw a line for each amplified parameter,
4. draw vertical lines from the key periods to individuate the connection ramps,
5. draw the idealised design spectrum.

A common assumption, used when only the $\ddot{u}_{g,0}$ estimate is available, is

$$\dot{u}_{g,0} = \ddot{u}_{g,0} \frac{120 \, \text{cm} \, \text{s}^{-1}}{g} \quad \text{and} \quad u_{g,0} = \ddot{u}_{g,0} \frac{90 \, \text{cm}}{g}.$$

Comparison of Design and Response Spectra, 1

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Example and Summary

In the figure, the response spectrum for the 1940 El Centro NS acceleration record, computed for $\zeta = 5\%$, and the corresponding design spectrum, with amplifications corresponding to median values of the ordinates.
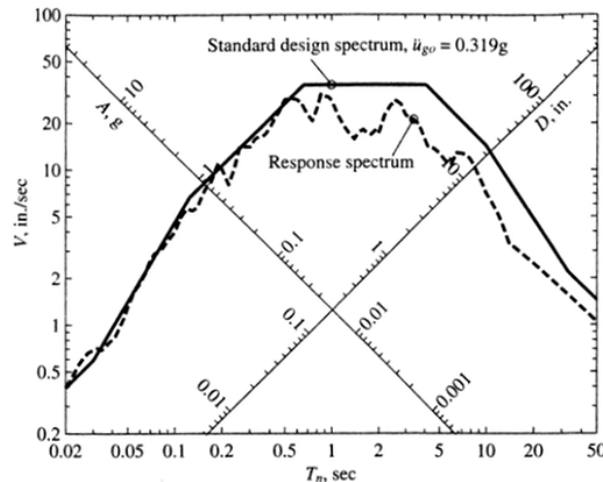
The spectrum was constructed from the real value of $\ddot{u}_{g,0} = 0.319\,g$ and estimated values of $\dot{u}_{g,0} = \ddot{u}_{g,0}\frac{48\,\text{inch/s}}{g} = 15.3\,\text{inch/s}$ and $u_{g,0} = \ddot{u}_{g,0}\frac{90\,\text{cm}}{g} = 11.5\,\text{inch}$, estimated values that are significantly higher than the effective values.

There is a good concordance in the acceleration controlled part of the design spectrum, but spectral velocities and deformations are not very good, due to rather poor estimates of the relevant ground motion peak quantities.

Comparison of Design and Response Spectra, 2

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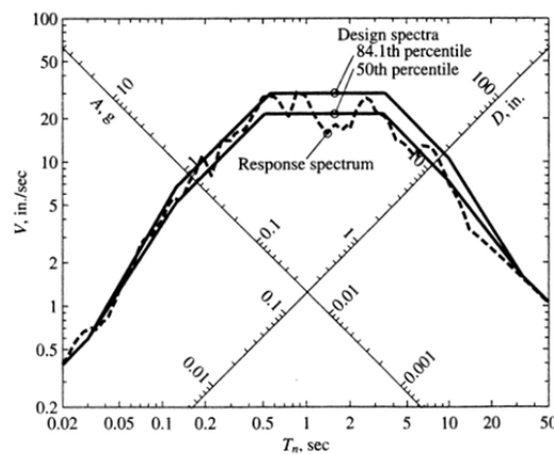Earthquakes and Earthquake Response

Response Spectrum

Response Spectrum Characteristics

Idealised Response Spectra
Elastic Design Spectra
Example and Summary

In this second slide the design spectra are two, the median and the median $+ 1\sigma$ versions, both based on *exact* peak values of the ground motion.

While the median spectrum is, ok, in a median position with respect to the ordinates of the elastic response spectrum, the presumed envelope spectrum does effectively a good job,

maxing out most of the spikes present in the elastic response spectrum.

Differences between Response and Design Spectra

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Example and
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The response spectrum is a description, in terms of its peak effects, of a particular ground motion.

The design spectrum is a specification, valid for a site or a class of sites, of design seismic forces.

If a site falls in two different classifications, e.g., the site is near to a seismic fault associated with low magnitude earthquakes and it is distant from a fault associated with high magnitude earthquakes, with the understanding that the frequency contents of the two classes of events are quite dissimilar the design spectrum should be derived from the superposition of the two design spectra.