Second Home Assignment

June 24, 2010

Contents

You must submit your paper before your oral examination.

1 Differential support motion

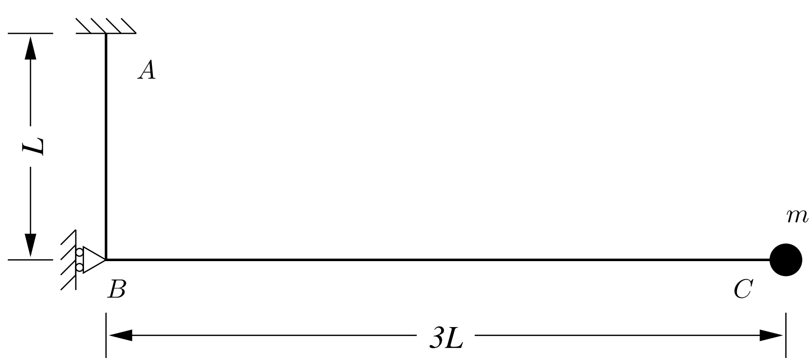

Figure 1: structural model

The system in figure 1 can be modeled as a SDOF system, disregarding the beam mass that is much less than the suspended mass M.

The system is excited by the horizontal motion of the external support in B (NB: the beam is continuous in B), namely $u_B = 3\text{mm} \sin \omega t$.

Assume that axial and shear deformations are negligible with respect to the flexural deformations.

1. Determine the natural frequency of vibration.

2. Determine the influence matrix E.
3. Write the equation of motion.
4. Find the peak value of the bending moment in A.

The relevant parameters are:

- $m = 18000$kg,
- $L = 2$m,
- $EJ = 3.6$MN m^2,
- $\omega = 3 \cdot 2\pi$,
- $\zeta = 0.03$.

2 Continuous system

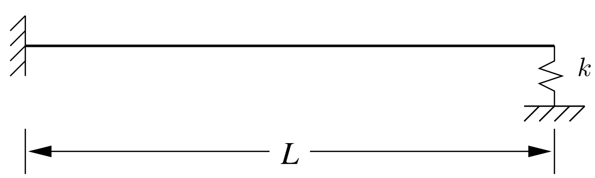

Figure 2: elastically supported beam

The beam in the figure 2 is clamped at the left end and it is supported by a spring at the right end, has constant unit mass m and constant flexural stiffness EJ.

Find the first and second frequencies of free vibration and plot the corresponding eigenfunctions for $k_{\text{spring}} = 2\frac{EJ}{L^3}$.

Find the first and second frequencies for $k_{\text{spring}} = \alpha\frac{EJ}{L^3}$ and plot in a log-lin diagram the eigenfrequencies $\omega_i(\alpha),\ i = 1, 2$ for $10^{-3} < \alpha < 10^{+3}$.

3 Inelastic design

A SDOF system has a natural frequency of vibration $f_n = 2.5$Hz and a yield strength $f_y = 0.60w$, where w is the system's weight.

Find the system's required ductility μ and its peak displacement x_m for a design maximum ground acceleration of 0.4g, knowing that the amplification factor for spectral accelerations (elastic) is $\alpha_A = 2.4$ and that the spectral region in which the system lies is the region where $A = \alpha_A \ddot{x}_{g0}$.

If we want to reduce by 10% the maximum displacement x_m, what should be done?