Dynamics of Structures 2009-2010

1st home assignment

Instructions

This assignment is due on Thursday 2010-06-03. You can check in your paper after the class, or at your choice you can submit a copy by email.

- Checked in submissions should be printed or, as an alternative, *nicely* handwritten.
- Email submissions should consist of
  - an optional introductory text,
  - a mandatory PDF[1] [2] attachment with your solutions to the problems, organized as explained below,
  - optional attachments with materials that you deem relevant, e.g., a spreadsheet or the source code of some program you wrote,
  - no Word files[3] please.

For each of the *six problems*, copy the text of the problem, *briefly* summarize the procedure you'll be using, detail all relevant steps including part of intermediate numerical results as you see fit, *clearly state* the required answers.

Weights are ≈ 10 20 20 35 35 30 for exact results and clean developments. Minimum required is 90.

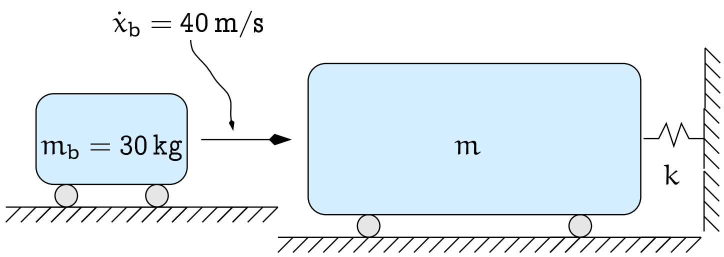
[1] By PDF, I mean something produced by LaTeX or Office or similar, ok? No scans (or photos!) of your handwriting, thank you.

[2] Please check that all needed fonts are included in the file before sending. In doubt, http://en.allexperts.com/q/Microsoft-Word-1058/2009/8/embedding-pdf-file.htm.

[3] Word files are rendered differently by different versions of Word, i.e., mine and yours. Word files are modifiable. No Word files, please

Your cooperation must be restricted to discussion of ideas and procedures, while actual developement of procedures and writing must be strictly individual. Beware: common work is very easy to spot.

1 Impact



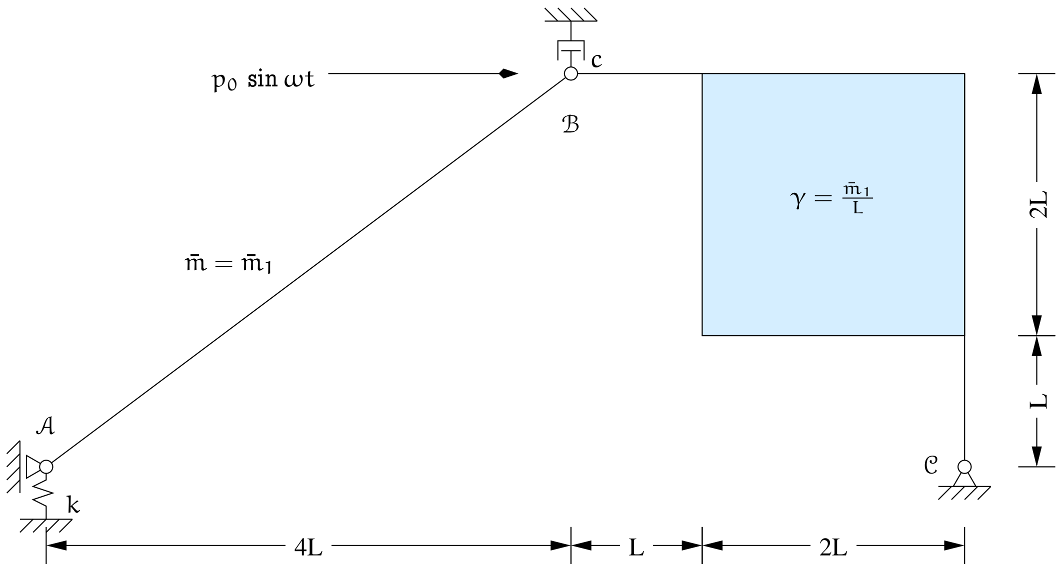
A body of mass $m_b = 30\,\text{kg}$ hits an undamped *SDOF* system, of unknown characteristics k and m, with velocity $\dot{x}_b = 40\,\text{m}\,\text{s}^{-1}$.

After collision the two masses are «glued» together and a measurement of the ensuing free oscillations give the following results:

$$x_{max} = 48\,\text{mm}, \qquad \dot{x}_{max} = 240\,\text{mm}\,\text{s}^{-1}.$$

What is the natural frequency of vibration of the *original* single degree of freedom system?

2 Generalised Coordinates (rigid bodies)



The articulated rigid system in figure is composed by two rigid bars,

- $\overline{\mathcal{AB}}$, with unit mass $\bar{m}_1$,
- the massless $\overline{\mathcal{BC}}$;

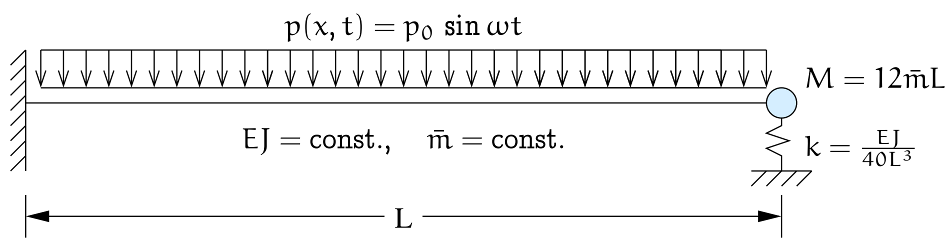
and by a rigid square body that is solidal to $\overline{\mathcal{BC}}$, with unit mass $\gamma = \bar{m}_1/L$.

The fixed constraints are a vertical roller in $\mathcal{A}$, a hinge in $\mathcal{C}$ and an internal hinge in $\mathcal{B}$, the deformable constraints are a vertical spring in $\mathcal{A}$, its stiffness $= k$ and a vertical dashpot in $\mathcal{B}$, its damping coefficient $= c$.

The system is excited by an horizontal force, $p(t) = p_o \sin \omega t$.

Using preferably the rotation of $\overline{\mathcal{AB}}$ about $\mathcal{A}$ as the generalised coordinate, write the equation of equilibrium of the system using the Principle of Virtual Displacements.

3 Generalised Coordinates (flexible systems)



The beam in figure, clamped at the left and supported by a spring at the right, supports a dimensionless body at the right end.

The bending stifness and unit mass of the beam, EJ and $\bar{m}$, are constants, the supported body has mass $M = 12\bar{m}L$ and the spring has stiffness $k = \frac{EJ}{40L^3}$.

The beam-mass-spring system is excited by a spatially uniform, distributed load $p(x, t) = p_0 \sin \omega t$.

Using an appropriate shape function write the equation of motion of the equivalent *SDOF* system.

4 Numerical Integration

A *SDOF* has the following characteristics:

$$\begin{aligned} k &= 32\,\mathrm{kN\,m^{-1}}, \\ m &= 1800\,\mathrm{kg}, \\ \zeta &= 7\%, \\ f_y &= 2.5\,\mathrm{kN} \end{aligned}$$

and is subjected to a loading $p(t)$,

$$p(t) = 30\,\mathrm{kN} \begin{cases} at + 12(at)^2 - 64(at)^3, & 0 \leqslant t \leqslant 0.25\,\mathrm{s}, \text{ with } a = 1\,\mathrm{s}^{-1}, \\ 0 & \text{otherwise.} \end{cases}$$

Disregarding the non-linear behaviour, for initial rest conditions, give the exact equation of motion, $x = x(t)$ and integrate numerically[4] the equation of motion with

1. the algorithm of central differences,
2. the algorithm of constant acceleration and
3. the algorithm of linear acceleration,

with time step $h = 0.005\,\mathrm{s}$ in all three cases.

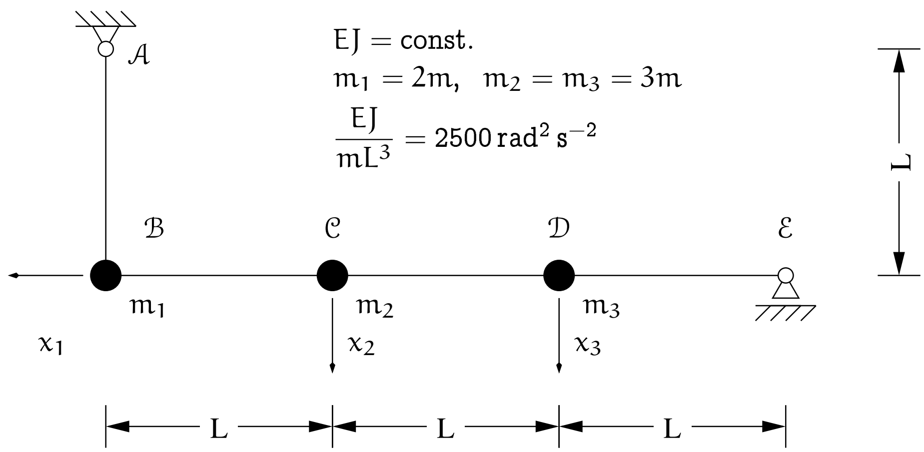
Plot the results of the numerical procedures and the exact solution.

Optional

Repeat the exercise keeping into account non-linear behaviour.

[4]Do not print all the intermediate results for every time step for every procedure.

5 *MDOF* System



In the structure depicted above, the structural mass is negligible with respect to the masses of the three supported bodies, so it is correct to use the three *DOF*'s in the figure as the dynamical degrees of freedom of the system.

1. Compute the flexibilty matrix[5] $\mathbf{F}$ and the mass matrix[6] $\mathbf{M}$.

2. Compute all the eigenvalues and the eigenvectors of the 3 *DOF* system using a method of your choice.

3. For non-null initial velocities

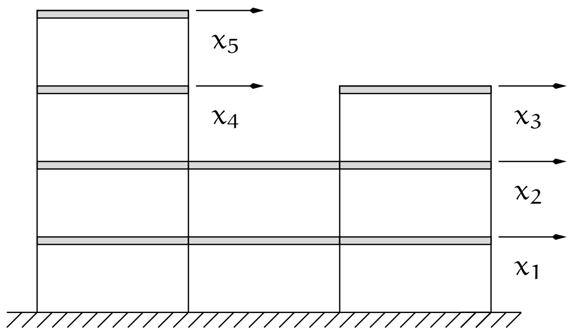
$$\dot{x}_0 = \begin{Bmatrix} 1 & 0 & 0 \end{Bmatrix}^T \frac{L}{2500\,\mathrm{s}}$$

   compute the rotations in $\mathcal{A}$.

[5]Disregarding axial deformability.

[6]Be careful, $m_{11} \neq m_1$!

6 Rayleigh-Ritz & Subspace Iteration



The structure depicted above can be analyzed as a shear type building. All columns are equal, each with a lateral stiffness indicated by k. One of the consequences of the previous statement is that the direct stiffness k_{11} is equal to $8k$, because the number of column that must be deformed to have a unit displacement in x_1 is 8.
The mass matrix is diagonal, with $m_{11} = m_{22} = 3m$ and $m_{33} = m_{44} = m_{55} = m$.

oOo

Find the first three eigenvalues and the first three eigenvectors of the structure using the Rayleigh-Ritz procedure with the Ritz base $\hat{\mathbf{\Phi}}_0$ indicated on the right, denoting the Ritz coordinates eigenvector matrix with $\mathbf{Z}$.

$$\hat{\mathbf{\Phi}}_0 = \begin{bmatrix} +1 & +1 & +1 \\ +2 & +1 & +1 \\ +3 & +0 & -1 \\ +3 & +0 & +1 \\ +4 & -1 & +1 \end{bmatrix}$$

oOo

Do one subspace iteration, deriving a new set of Ritz base vectors,

$$\hat{\mathbf{\Phi}}_1 = \mathbf{K}^{-1}\mathbf{M}\mathbf{\Phi}\mathbf{Z}_0.$$

oOo

Find the first three eigenvalues and the first three eigenvectors of the structure using the Rayleigh-Ritz procedure with the Ritz base $\hat{\mathbf{\Phi}}_1$.

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Discuss the two set of results.