Dynamics of Structures 2009-2010

1st home assignment

Instructions

This assignment is due on Tuesday 7th of September. You must check in your assignment by email, in the form of a PDF[1] [2] attachment with your solutions to the problems (no MS Word files, thank you)

For each of the *six problems*, copy the text of the problem, *briefly* summarize the procedure you'll be using, detail all relevant steps including part of intermediate numerical results as you see fit, *clearly state* the required answers.

Weights are $\approx$ 10 20 20 35 35 30 for exact results and clean developments. Minimum required is 90.

Your cooperation must be restricted to discussion of ideas and procedures, while actual developement of procedures and writing must be strictly individual. Beware: common work is very easy to spot.

1 Dynamical Testing

A simply supported beam is loaded at mid-span with a harmonic load, with different frequencies and the same *nominal*load amplitude.

Due to the position of the load and the frequencies of excitation used (close to the natural frequency, as determined by preliminary tests), it is reasonable to assume that only the first mode of vibration of the beam is excited.

[1] By PDF, I mean something produced by LaTeX or Office or similar.

[2] Please check that all needed fonts are included in the file before sending. In doubt, http://en.allexperts.com/q/Microsoft-Word-1058/2009/8/embedding-pdf-file.htm.

The steady-state response parameters (amplitude and phase difference) are measured for each loading frequency, and are here reported in table 1. Determine the characteristics of the first mode of vibration of the beam.

f [Hz]	p_0 [N]	Δ_{max} [mm]	θ [deg]
3.40	1200	5.811	6.879
3.80	1200	9.143	11.863
4.20	1200	21.296	32.012
4.60	1200	23.079	141.294
5.00	1200	8.450	165.476
5.40	1200	4.972	171.082

Table 1: load and response characteristics

Be warned that all the data reported in table 1 is affected by experimental errors, so that particular care should be used to get the best possible estimate of ω and ζ.

2 Vibration Isolation

A rotating machine weights 75kN and during operation transmits to its rigid foundation a harmonic load of 2.4kN, at a frequency f=60Hz.

It is intended to reduce the transmitted force to 500N, suspending the machine over a system of elastic supports.

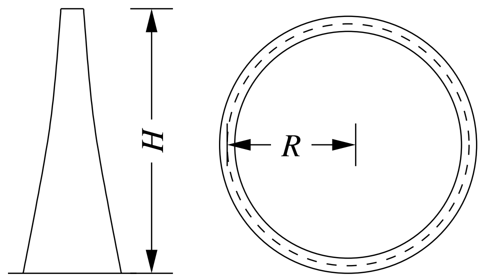
1. What should be the total stiffness of the elastic supports to achieve the required tranmitted force reduction?

2. What if the dynamic displacements that the machine will experience when mounted on the elastic support are too large for a correct operation? Is it possible to modify the support system so that the displacements are reduced and the harmonic force transmitted still is no greater than 500N?

3 Generalised Coordinates (flexible systems)

Estimate the natural frequency of vibration of a tower-like structure, characterized by an hollow, annular cross section with constant thickness t =

0.32 m, mean radius $R = R(x) = \exp(-0.28768\frac{x}{H})3.2$ m, height $H = 72$ m, Young modulus $E = 30$ GPa, density $\rho = 2500\,\mathrm{kg\,m^{-3}}$



4 Numerical Integration

A single degree of freedom system, with a mass $m = 120$ kg, a stiffness $k = 125\,\mathrm{kN\,m^{-1}}$ and a damping ratio $\zeta = 0.12$ is at rest when it is subjected to an external force $p(t)$:

$$p(t) = \begin{cases} (4000(at)^3 - 1280(at)^2 + 96at)\mathrm{kN} & \text{for } 0.0 \leqslant t \leqslant 0.20\mathrm{s}, \\ 0.0 & \text{otherwise,} \end{cases}$$

where $a = 1\,\mathrm{s^{-1}}$.

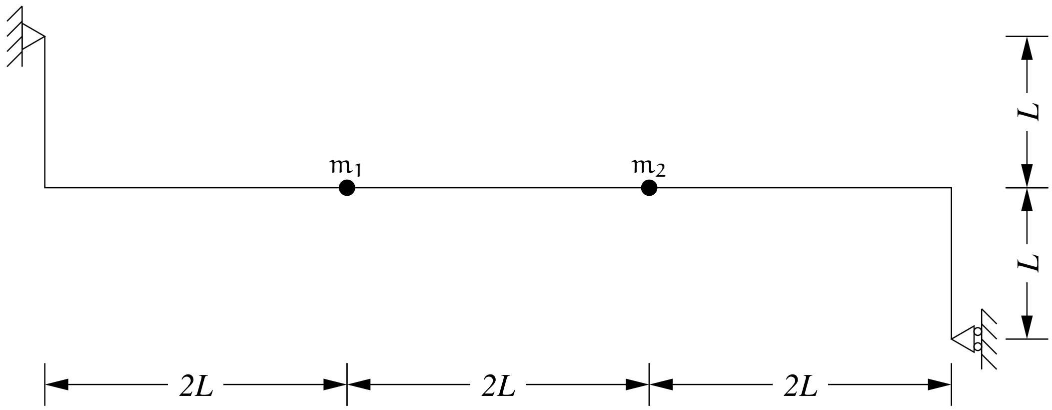
1. Find the exact response in the time interval $0 \leqslant t \leqslant 0.5$ s, using superposition of the general integral and an appropriate particular solution.

2. Integrate the equation of motion numerically using the algorithm of constant acceleration, with two different integration steps $h_1 = 0.02$s and $h_2 = 0.005$s, in the same time interval.

3. As above, this time using the linear acceleration algorithm.

4. Plot your results in a meaningful manner and comment your results.

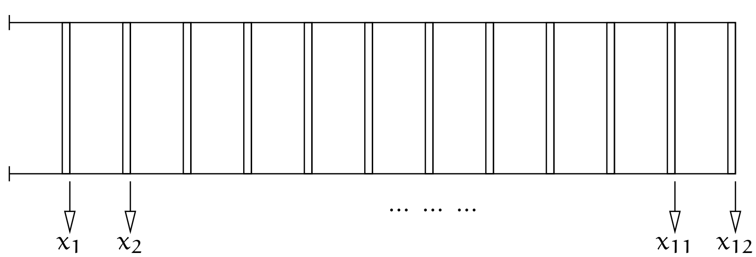
5 3 *DOF* System



An uniform, simply supported beam substains two equal point masses, the beam mass being negligible with respect to the substained mass. Consider negligible also the axial and shear deformations of the beam.

1. Write the structural matrices, find the eigenvalues solving the determinantal equation, find the eigenvectors of the system.
2. Examine the behaviour of the system when its supports are subjected to a horizontal earthquake excitation, $\ddot{u}_g = \ddot{u}_g(t)$.

6 Rayleigh-Ritz & Subspace Iteration



The structure above can be analyzed as a shear type building. The mass matrix is diagonal, with storey masses being all equal to m. The storey stiffnesses are linearly decreasing, $k_1 = 23k$, $k_2 = 22k$, ..., $k_{11} = 13k$ and $k_{12} = 12k$. For example, the stifness matrix element $k_{9,9}$ is given by $k_{9,9} = k_9 + k_{10} = 29k$.

1. Find the lower four eigenvalues and eigenvectors of the structure[3] using the Rayleigh-Ritz procedure, denoting the Ritz base with $\hat{\mathbf{\Phi}}_0$ and the Ritz coordinates eigenvector matrix with Z.

2. Do one subspace iteration, deriving a new set of Ritz base vectors,

$$\hat{\mathbf{\Phi}}_1 = \mathsf{K}^{-1}\mathsf{M}\Phi\mathsf{Z}_0.$$

3. Find the lower four eigenvalues and eigenvectors of the structure using the Rayleigh-Ritz procedure with the Ritz base $\hat{\mathbf{\Phi}}_1$.

4. Discuss the two set of results.

[3]The eigenvectors ψ_i of the structure are different from the eigenvectors in Ritz coordinates.