Earthquake Response of Inelastic Systems

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Motivation

Cyclic Behaviour

E-P Idealisation

EQ Response of E-P Systems

Effects of Yielding

Outline

Motivation

Cyclic Behaviour of Structural Members

Elasto-plastic Idealisation

EQ Response of E-P Systems
- Normalised Equation of Motion

Effects of Yielding
- Inelastic Response, different values of $\bar{f}_y$

Motivation

In Earthquake Engineering it is common practice to design against a quake that has a given mean period of return (say 500 years), quite larger than the expected life of the construction.

However, in the almost totality of cases the structural engineer does not design the anti-seismic structures considering the ordinates of the elastic spectrum of the maximum earthquake, the preferred procedure is to reduce these ordinates by factors that can be as high as 6 or 8.

This, of course, leads to a large reduction in the cost of the structure.

The reasoning behind such design procedure is that, for an unlikely occurrence of a large eq, a large damage in the construction is acceptable as far as no human lives are taken in a complete structural collapse and that, in the mean, the costs for repairing a damaged building are not disproportionate to its value.

What to do?

To ascertain the amount of acceptable reduction of eq loads it is necessary to study

- the behaviour of structural members and systems subjected to cyclic loading outside the elastic range, to understand the amount of plastic deformation and cumulated plastic deformation that can be sustained before collapse and
- the global structural behaviour for inelastic response, so that we can relate the reduction in design ordinates to the increase in members plastic deformations.

The first part of this agenda pertains to Earthquake Engineering proper, the second part is across EE and Dynamics of Structures, and what you will see today.

Cyclic behaviour

Investigation of the cyclic behaviour of structural members, sub-assemblages and scaled or real sized building model, either in labs or via numerical simulations, constitutes the bulk of EE.

What is important, at the moment, is the understanding of how different these behaviours can be, due to different materials or structural configurations, with instability playing an important role.

We will see 3 different diagrams, force vs deformation, for a clamped steel beam subjected to flexure, a reinforced concrete sub-assemblage an a masonry wall.

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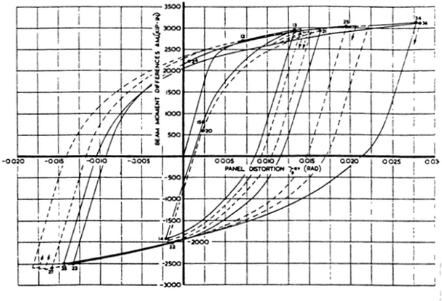

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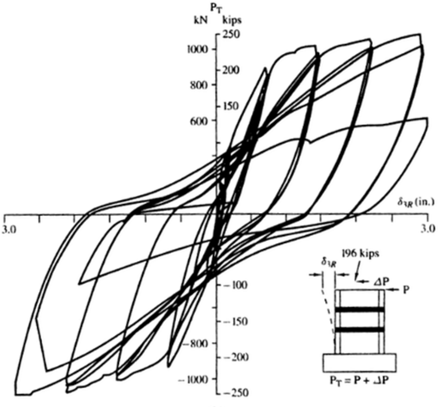

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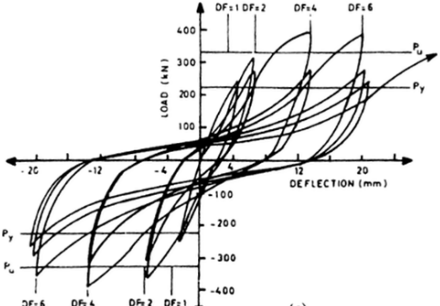

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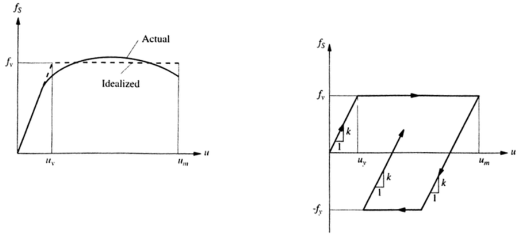
E-P model



A more complex behaviour may be represented with an elasto-perfectly plastic (e-p) bilinear idealisation, see figure, where two important requirements are obeyed

1. the initial stiffness of the idealised e-p system is the same of the real system, so that the natural frequencies of vibration for small deformation are equal,
2. the yielding strength is chosen so that the sum of stored and dissipated energy in the e-p system is the same as the energy stored and dissipated in the real system.

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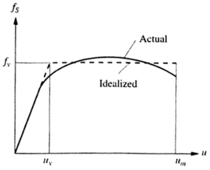

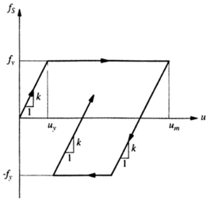

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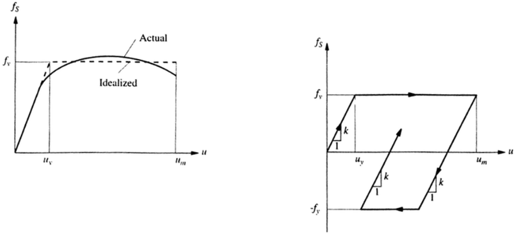
E-P model, 2



In perfect plasticity, when yielding (a) the force is constant, $f_S = f_y$ and (b) the stiffness is null, $k_y = 0$. The force f_y is the yielding force, the displacement $x_y = f_y/k$ is the yield deformation.
In the right part of the figure, you can see that at unloading $(\,dx = 0)$ the stiffness is equal to the initial stiffness, and we have $f_s = k(x - x_{p_{tot}})$ where $x_{p_{tot}}$ is the total plastic deformation.

Definitions

For a given seismic excitation, we give the following definitions

equivalent system a linear system with the same characteristics $(\omega_n,\ \zeta)$ of the non-linear system

normalised yield strength, $\bar{f}_y$ is the ratio of the yield strength to the peak force of the equivalent system,

$$\bar{f}_y = \min\left\{\frac{f_y}{f_0} = \frac{x_y}{x_0}, 1\right\}.$$

It is $\bar{f}_y \leqslant 1$ because for $f_y \geqslant f_0$ there is no yielding, and in such case we define $\bar{f}_y = 1$.

yield strength reduction factor, R_y it comes handy to define R_y, as the reciprocal of $\bar{f}_y$,

$$R_y = \frac{1}{\bar{f}_y} = \max\left\{1, \frac{f_0}{f_y} = \frac{x_0}{x_y}\right\}.$$

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Definitions, cont.

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e-p peak response, x_m the elasto-plastic peak response

$$x_m = \max_t \{|x(t)|\}.$$

ductility factor, μ (or ductility ratio) the normalised value of the e-p peak response

$$\mu = \frac{x_m}{x_y}.$$

Whenever it is $R_y > 1$ it is also $\mu > 1$.

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Note that the ratio between the e-p and elastic peak responses is given by

$$\frac{x_m}{x_0} = \frac{x_m}{x_y}\frac{x_y}{x_0} = \mu\,\bar{f}_y = \frac{\mu}{R_y} \;\rightarrow\; \mu = R_y\frac{x_m}{x_0}.$$

Normalising the force

For an e-p system, the equation of motion (*EOM*) is

$$m\ddot{x} + c\dot{x} + f_S(x, \dot{x}) = -m\ddot{u}_g(t)$$

with f_S as shown in a previous slide. The *EOM* must be integrated numerically to determine the time history of the e-p response, $x(t)$.
For a given excitation $\ddot{x}_g(t)$, the response depends on 3 parameters, $\omega_n = \sqrt{k/m}$, $\zeta = c/(2\omega_n m)$ and x_y.
If we divide the *EOM* by m, recalling our definition of the normalised spring force, the last term is

$$\frac{f_S}{m} = \frac{1}{m}\,\frac{f_y}{f_y}\,f_S = \frac{1}{m}\,k\,x_y\,\frac{f_S}{f_y} = \omega_n^2 x_y \bar{f}_S$$

and we can write

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x_y \bar{f}_S(x, \dot{x}) = -\ddot{u}_g(t)$$

Normalising the displacements

With the position $x(t) = \mu(t)\,x_y$, substituting in the *EOM* and dividing all terms by x_y, it is

$$\ddot\mu + 2\omega_n\zeta\dot\mu + \omega_n^2\,\bar f_S(\mu,\dot\mu) = -\frac{\omega_n^2}{\omega_n^2}\frac{\ddot x_g}{x_y} = -\omega_n^2\frac{\ddot x_g}{a_y}$$

It is now apparent that the input function for the ductility response is the acceleration ratio: doubling the ground acceleration or halving the yield strength leads to exactly the same response $\mu(t)$ and the same peak value μ.

The equivalent acceleration can be expressed in terms of the normalised yield strength $\bar f_y$,

$$a_y = \frac{f_y}{m} = \frac{\bar f_y\, f_0}{m} = \frac{\bar f_y\, kx_0}{m} = \bar f_y\,\omega_n^2\,x_0$$

and recognising that x_0 depends only on ζ and ω_n we conclude that, for given $\ddot x_g(t)$ and $\bar f_S(\mu,\dot\mu)$ the ductility response depends only on ζ, ω_n, $\bar f_y$.

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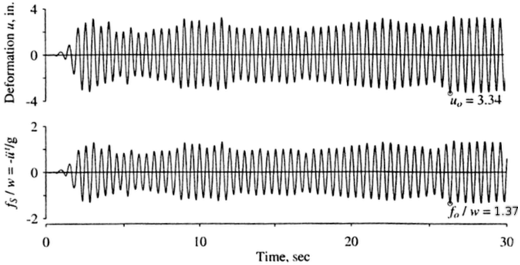
Elastic response, required parameters

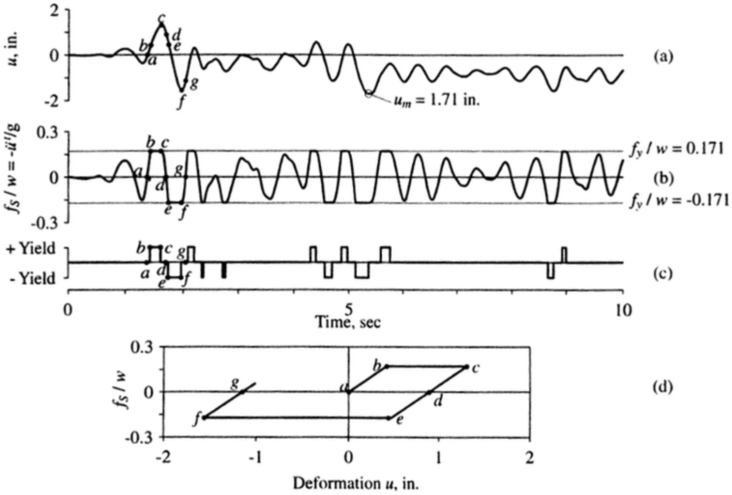


Response of linear system with $T_n = 0.5$ sec and $\zeta = 0$ to El Centro

In the figure above, the elastic response of an undamped, $T_n = 0.5\,\mathrm{s}$ system to the NS component of the El Centro 1940 ground motion (all our examples will be based on this input motion).

Top, the deformations, bottom the elastic force normalised with respect to weight, from the latter peak value we know that all e-p systems with $f_y < 1.37w$ will experience plastic deformations during the EC1940NS ground motion.

Inelastic response, $\bar{f}_y = 1/8$



The various response graphs above were computed for $\bar{f}_y = 0.125$ (i.e., $R_y = 8$ and $f_y = \frac{1.37}{8} w = 0.171 w$) and $\zeta = 0$, $T_n = 0.5\,\mathrm{s}$.

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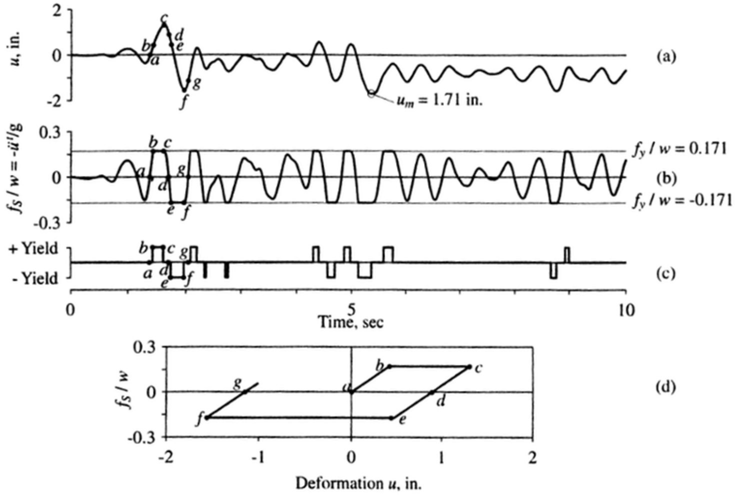

Top, the deformation response, note that the peak response is $x_m = 1.71\,\text{in}$, different from $x_0 = 3.34\,\text{in}$; it is $\mu = R_y \frac{x_m}{x_0} = 4.09$

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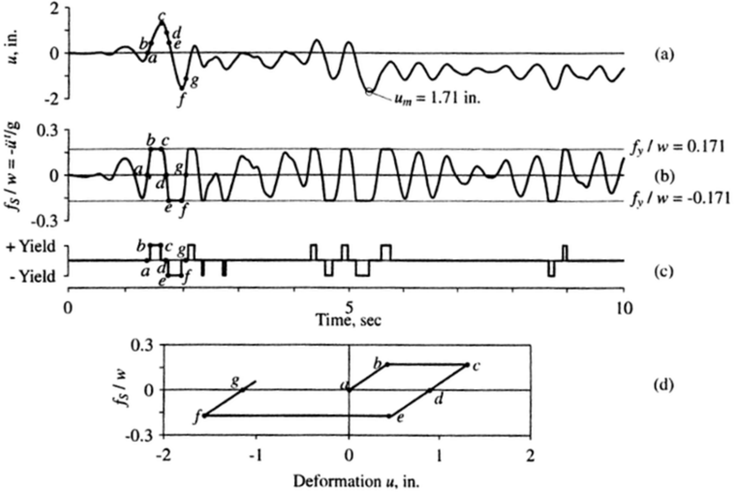
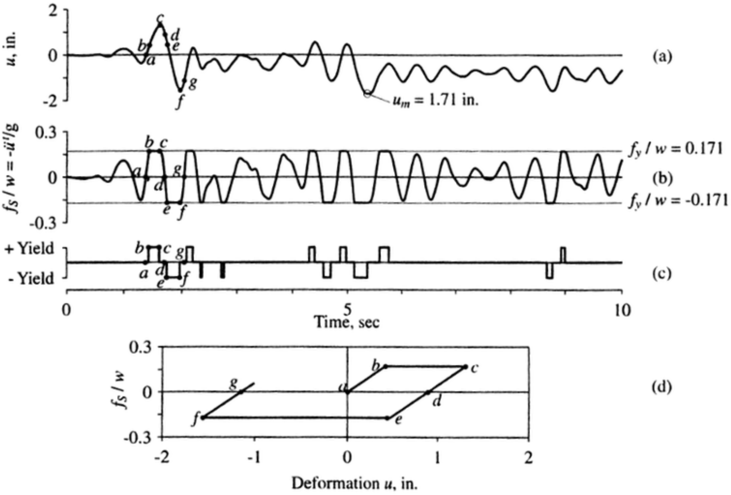


Second row, normalised force f_S/w, note that the response is *clipped* at $f_S = f_y = 1.171w$

Inelastic response, $\bar{f}_y = 1/8$



Third row, response in terms of yielding state, positive or negative depending on the sign of velocity

Inelastic response, $\bar{f}_y = 1/8$

The force-deformation diagram for the first two excursions in plastic domain, the time points a, b, c, d, e, f and g are the same in all 4 graphs:

- until $t = b$ we have an elastic behaviour,
- until $t = c$ the velocity is positive and the system accumulates positive plastic deformations,
- until $t = e$ we have an elastic unloading (note that for $t = d$ the force is zero, the deformation is equal to the total plastic deformation),
- until $t = f$ we have another plastic excursion, cumulating negative plastic deformations
- until at $t = f$ we have an inversion of the velocity and an elastic reloading.

Response for different $\bar{f}_y$'s

$\bar{f}_y$	x_m	x_{perm}	μ
1.000	2.25	0.00	1.00
0.500	1.62	0.17	1.44
0.250	1.75	1.10	3.11
0.125	2.07	1.13	7.36

This table was computed for $T_n = 0.5\,\text{s}$ and $\zeta = 5\%$ for the EC1940NS excitation.

Elastic response was computed first, with peak response $x_0 = 2.25\,\text{in}$ and peak force $f_0 = 0.919w$, later the computation was repeated for $\bar{f}_y = 0.5,\ 0.25,\ 0.125$.

In our example, all the peak values of the e-p responses are smaller than the elastic one, but this is not always true, and shouldn't be generalised.

The permanent displacements increase for decreasing yield strengths, and also this fact shouldn't be generalised.

Last, the ductility ratios increase for decreasing yield strengths, for our example it is $\mu \approx R_y$.

Ductility demand and capacity

We can say that, for a given value of the normalised yield strength $\bar{f}_y$ or of the yield strength reduction factor R_y, there is a *ductility demand*, a measure of the extension of the plastic behaviour that is required when we reduce the strength of the construction.

Corresponding to this ductility demand our structure must be designed so that there is a sufficient *ductility capacity*.

Ductility capacity is, in the first instance, the ability of individual members to sustain the plastic deformation demand without collapsing, the designer must verify that the capacity is greater than the demand for all structural members that go non linear during the seismic excitation.

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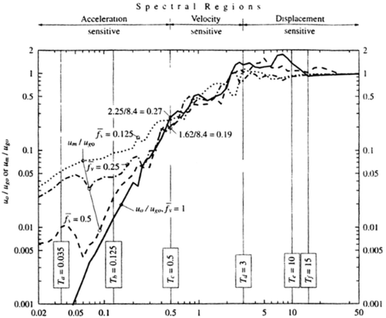
Effects of T_n

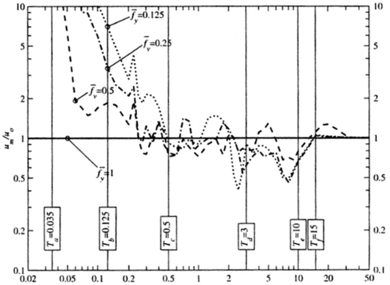


$x_m/x_{g,0}$

For EC1940NS, for $\zeta = .05$, for different values of T_n and for $\bar{f}_y = 1.0,\ 0.5,\ 0.25,\ 0.125$ the peak response x_0 of the equivalent system (in black) and the peak responses of the 3 inelastic systems has been computed.

There are two distintinct zones: left there is a strong dependency on $\bar{f}_y$, the peak responses grow with R_y; right the 4 curves intersects with each other and there is no clear dependency on $\bar{f}_y$.

Effects of T_n

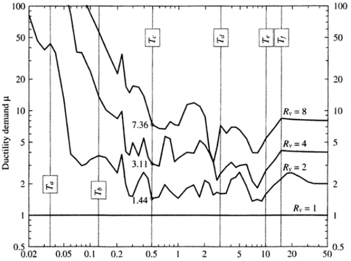


x_m/x_0

With the same setup as before, here it is the ratio of the x_m's to x_0, what is evident is the fact that, for large T_n, this ratio is equal to 1... this is justified because, for large T_n's, the mass is essentially at rest, and the deformation, either elastic or elasto-plastic, are equal and opposite to the ground displacement.

Also in the central part, where elastic spectrum ordinates are dominated by the ground velocity, there is a definite tendency for the x_m/x_0 ratio, that is $x_m/x_0 \approx 1$

Effects of T_n



μ

With the same setup as before, in this graph are reported the values of the ductility factor μ. The values of μ are almost equal to R_y for large values of T_n, and in the limit, for $T_n \to \infty$, there is a strict equality. An even more interesting observation regard the interval $T_c \leqslant T_n \leqslant T_f$, where the values of μ oscillate near the value of R_y.

On the other hand, the behaviour is completely different in the acceleration controlled zone, where μ grows very fast, and the ductility demand can result very high even for low values (0.5) of the yield strength reduction factor.

The results we have discussed are relative to one particular excitation, nevertheless research and experience confirmed that these propositions are true also for different EQ records, taking into account the differences in the definition of spectral regions.

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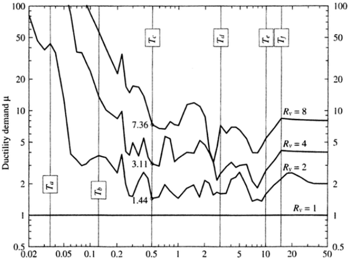

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Response Spectrum for Yield States

The first step in an anti seismic design is to set an available ductility (based on materials, conception, details).
In consequence, we desire to know the yield displacement u_y or the yield force f_y

$$f_y = k u_y = m \omega_n^2 u_y$$

for which the ductility demand demand imposed by the ground motion is not greater than the available ductility.

Response Spectrum for Yield States

For each T_n, ζ and μ, the *Yeld-Deformation Response Spectrum* (D_y) ordinate is the corresponding value of u_y: $D_y = u_y$. Following the ideas used for Response and Design Spectra, we define $V_y = \omega_n u_y$ and $A_y = \omega_n^2 u_y$, that we will simply call pseudo-velocity and pseudo-acceleration spectra.

Using our definitions of pseudo acceleration, we can find a more significant expression for the design force:

$$f_y = k u_y = m\omega_n^2 u_y = m A_y = w\frac{A_y}{g},$$

where w is the weight of the structure.

Our definition of inelastic spectra is compatible with the definition of elastic spectra, because for $\mu = 1$ it is $u_y = u_0$. Finally, the D_y spectrum and its derived pseudo spectra can be plotted on the tripartite log-log graph.

Response Spectrum for Yield States

For each T_n, ζ and μ, the *Yeld-Deformation Response Spectrum* (D_y) ordinate is the corresponding value of u_y: $D_y = u_y$. Following the ideas used for Response and Design Spectra, we define $V_y = \omega_n u_y$ and $A_y = \omega_n^2 u_y$, that we will simply call pseudo-velocity and pseudo-acceleration spectra.

Using our definitions of pseudo acceleration, we can find a more significant expression for the design force:

$$f_y = k u_y = m \omega_n^2 u_y = m A_y = w \frac{A_y}{g},$$

where w is the weight of the structure.

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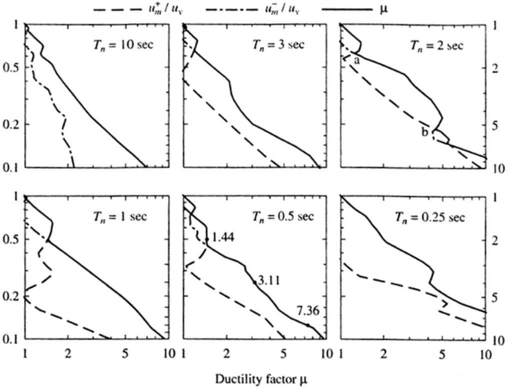
Computing D_y



On the left, for different T_n's and $\mu = 5\%$. the indipendent variable is in the ordinates, either $\bar{f}_y$ (left) or R_y (rigth) the strength reduction factor. Dash-dash lines is u_m^+/u_y, dash-dot is u_m^-/u_y.

u_m^+ and u_m^- are the peaks of positive and negative displacements of the inelastic system, the maximum of their ratios to u_y is the ductility μ.

If we look at these graphs using μ as the indipendent variable, it is possible that for a single value of μ there are different values on the tick line: in this case, for security reasons, the designer must design for the higher value of $\bar{f}_y$.

Example

For EC1940NS, $z = 5\%$, the yield-strength response spectra for $\mu = 1.0,\ 1.5,\ 2.0,\ 4.0,\ 8.0$.

On the left, a lin-lin plot of the pseudo-acceleration normalized (and adimensionalised) with respect to g, the acceleration of gravity.

On the right, a log-log tripartite plot of the same spectrum. Even a small value of μ produces a significant reduction in the required strength.

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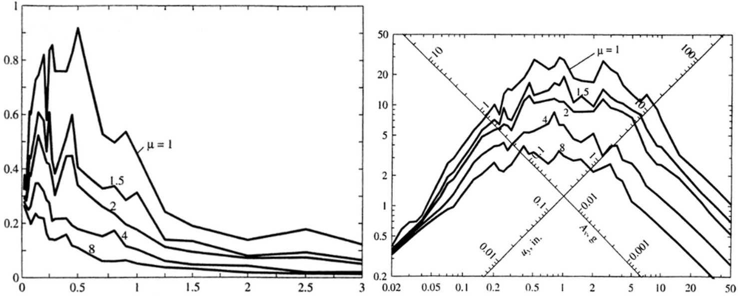

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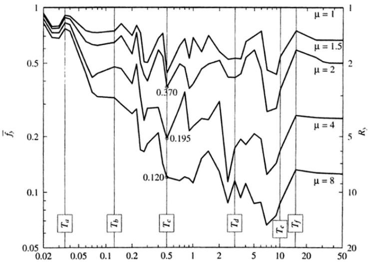
$\bar{f}_y$ vs μ

We have seen that $\bar{f}_y = \bar{f}_y(\mu, T_n, \zeta)$ is a monotonically increasing function of μ for fixed T_n and ζ.

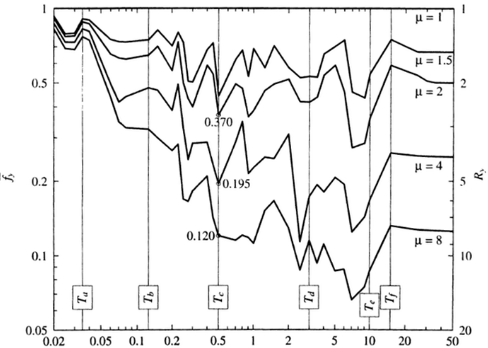


Left the same spectra of the previous slides, plotted in a different format, $\bar{f}_y$ vs T_n for different values of μ.

The implication of this figure is that an anti seismic design can be based on strength, ductility or a combination of the two.

For $T_n = 1.0$, the peak force for EC1940NS in an elastic sistem is $f_0 = 0.919\,w$, so it is possible to design for $\mu = 1.0$, hence $f_y = 0.919\,w$ or for an high value of ductility, $\mu = 8.0$, hence $f_y = 0.120 \cdot 0.919\,w$ or, if such an high value of ductility cannot be easily reached, design for $\mu = 4.0$ and a yielding force of 0.195 times f_0.

Yelding and Damping



El Centro 1940 NS, elastic response spectra and inelastic spectra for $\mu = 4$ and $\mu = 8$, for different values of ζ (2%, 5% and 10%).
Effects of damping are relatively important and only in the velocity controlled area of the spectra, while effects of ductility are always important except in the high frequency range.

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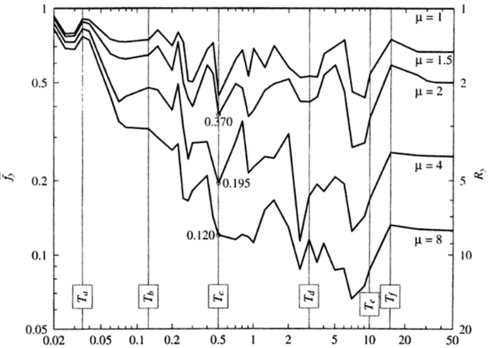

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Energy Dissipation

$$\int^{x(t)} m\ddot{x}\,dx + \int^{x(t)} c\dot{x}\,dx + \int^{x(t)} f_S(x,\dot{x})\,dx = -\int^{x(t)} m\ddot{x}_g$$

This is an energy balance, between the input energy $\int m\ddot{x}_g$ and the sum of the kinetic, damped, elastic and dissipated by yielding energy.

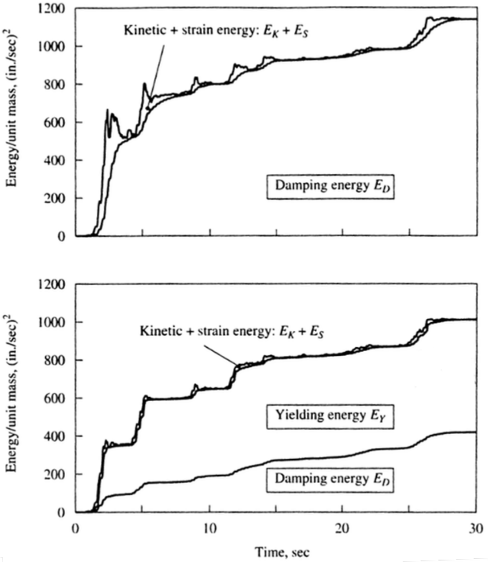
In every moment, the elastic energy $E_S(t) = \frac{f_S^2(t)}{2k}$ so the yielded energy is

$$E_y = \int f_S(x,\dot{x})\,dx - \frac{f_S^2(t)}{2k}.$$

The damped energy can be written as a function of t, as $dx = \dot{x}\,dt$:

$$E_D = \int c\,\dot{x}^2(t)\,dt$$

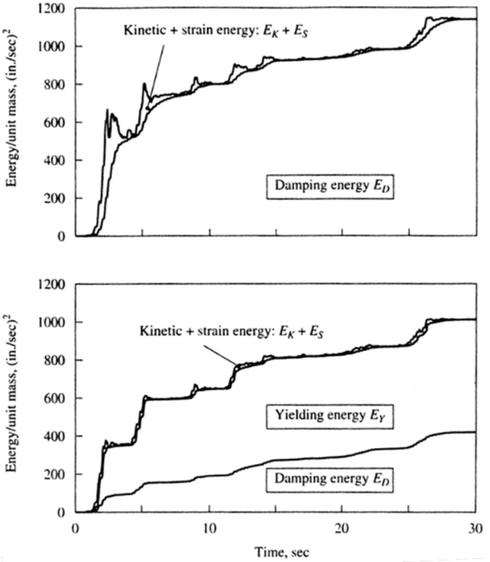
Energy Dissipation



For a system with $m = 1$ and
a) $\bar{f}_y = 1$
b) $\bar{f}_y = 0.25$
the energy contributions during the EC1940NS, $T_n = 0.5\,\text{s}$ and $\zeta = 5\%$.

In a), input energy is stored in kinetic+elastic energy during strong motion phases and is subsequently dissipated by damping.

In b), yielding energy is dissipated by means of some structural damage.



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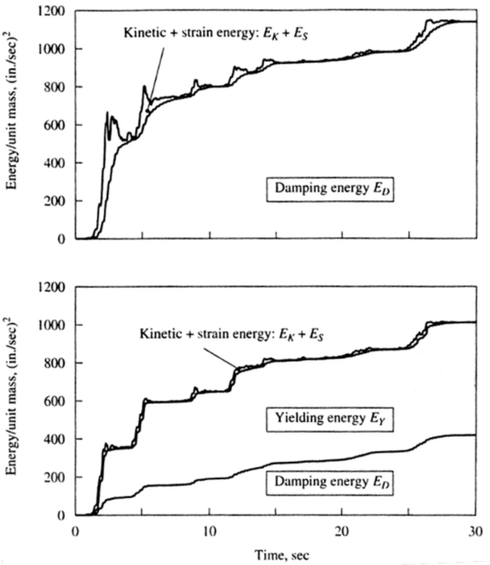

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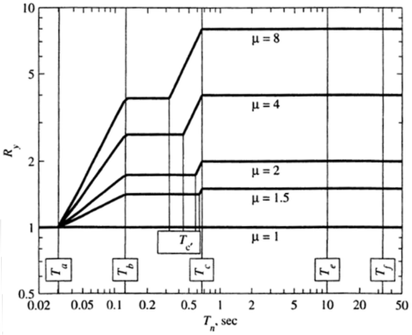
Inelastic Design Spectra

Two possible approaches:

1. compute response spectra for constant ductility demand for many consistent records, compute response parameters statistics and derive inelastic design spectra from these statistics, as in the elastic design spectra procedures;
2. directly modify the elastic design spectra to account for the ductility demand values.

The first procedure is similar to what we have previously seen, so we will concentrate on the second procedure, that it is much more used in practice.

R_y — μ — T_n equations

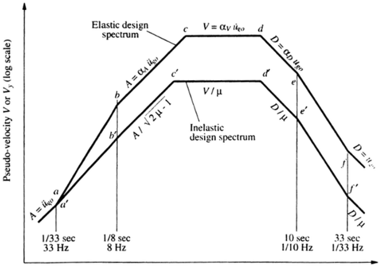


Based on observations and energetic considerations, the plots of R_y vs T_n for different μ values can be approximated with straight lines in a log-log diagram, where the constant pieces are defined in terms of the key periods in $D - V - A$ graphs.

$$R_y = \begin{cases} 1 & T_n < T_a \\ \sqrt{2\mu - 1} & T_b < T_n < T_{c'} \\ \mu & T_c < T_n \end{cases}$$

The key period $T_{c'}$ is different from T_c, as we will see in the next slide; the costant pieces are joined with straight lines in the log-log diagram.

Construction of Design Spectrum



Start from a given elastic design spectrum, defined by the points a-b-c-d-e-f.
Choose a value μ for the ductility demand.
Reduce all ordinates right of T_c by the factor μ, reduce the ordinates in the interval $T_b < T_n < T_c$ by $\sqrt{2\mu/1}$.

Draw the two lines $A = \frac{\alpha_A \ddot{x}_{g0}}{\sqrt{2\mu - 1}}$ and $A = \frac{\alpha_V \dot{x}_{g0}}{\mu}$, their intersection define the key point $T_{c'}$.
Connect the point $(T_a, A = \ddot{x}_{g0})$ and the point $(T_b, A = \frac{\alpha_V \dot{x}_{g0}}{\mu})$ with a straight line.
As we already know (at least in priciples) the procedure to compute the elastic design spectra for a given site from the peak values of the ground motion, using this simple procedure it is possible to derive the inelastic design spectra for any ductility demand level.

Important Relationships

For different zones on the T_n axis, the simple relationships we have previuosly defined can be made explicit using the equations that define R_y, in particular we want relate u_m to u_0 and f_y to f_0 for the elastoplastic system and the equivalent system.

1. region $T_n < T_a$, here it is $R_y = 1.0$ and consequently

$$u_m = \mu u_0 \qquad f_y = f_0.$$

2. region $T_b < T_n < T_{c'}$, here it is $R_y = \sqrt{2\mu - 1}$ and

$$u_m = \frac{\mu}{\sqrt{2\mu-1}} u_0 \qquad f_y = \frac{f_0}{\sqrt{2\mu-1}}$$

3. region $T_c < T_n$, here it is $R_y = \mu$ and

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Similar equations can be established also for the inclined connection segments in the R_y vs T_n diagram.

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Application: design of a SDOF system

- Decide the available ductility level μ (type of structure, materials, details etc).
- Preliminar design, m, k, ζ, ω_n, T_n.
- From an inelastic design spectrum, for known values of ζ, T_n and μ read A_y.
- The design yield strenght is

$$f_y = mA_y.$$

- The design peak deformation, $u_m = \mu D_y/R_y$, is

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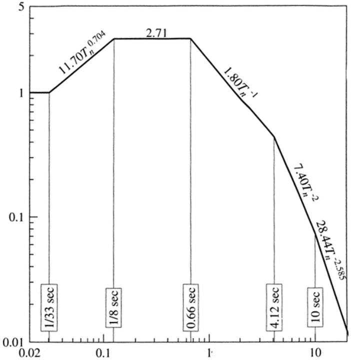

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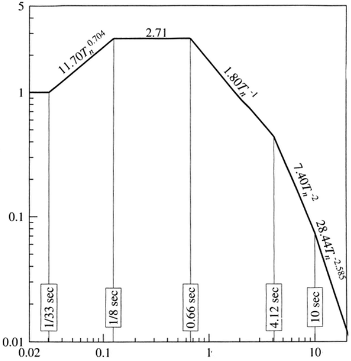

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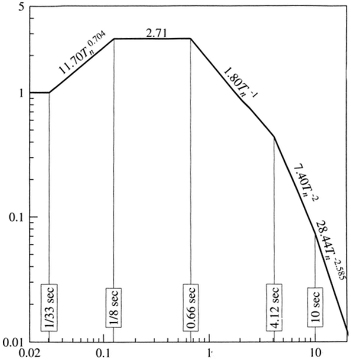

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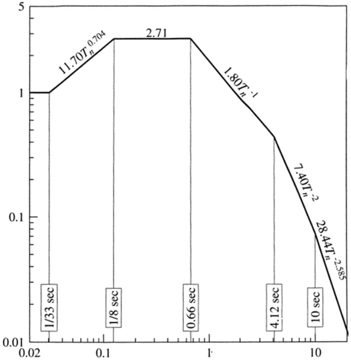

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