Dynamics of Structures 2010-2011

1st home assignment due on Tuesday 2011-06-17

Contents

Instructions

For each of the *six problems*, copy the text of the problem, *briefly* summarize the procedure you'll be using, detail all relevant steps including part of intermediate numerical results as you see fit, *clearly state* the required answers. Scores are $\approx 10, 20, 15, 15, 15, 25$.

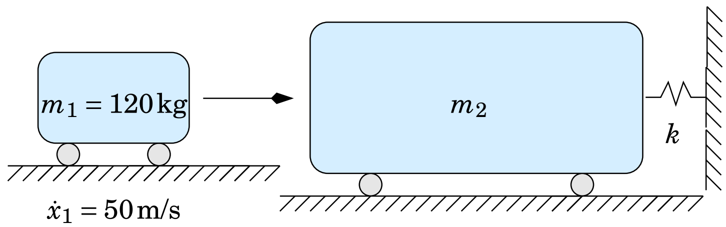
Submit your homework by email before 14:30 or, if you prefer, hand in your homework directly to me before the class.

- Handed in submissions must be printed or *nicely* handwritten.
- Email submissions *must include* a PDF[1] [2] attachment with your solutions. You can attach everything else is relevant for you (spreadsheets, your programs' sources) except Word files as I always encounter strange compability problems.

[1] a PDF as produced by Office or LaTeX or similar...no scans nor photos.

[2] Please check that all needed fonts are included in the file before sending. In doubt, http://en.allexperts.com/q/Microsoft-Word-1058/2009/8/embedding-pdf-file.htm.

1 Impact



A body of mass $m_1 = 120\,\mathrm{kg}$ hits an undamped *SDOF* system, of unknown characteristics k and m_2, with velocity $\dot{x}_1 = 50\,\mathrm{m\,s^{-1}}$.

The collision is anelastic, i.e., the two masses are *glued* together and a measurement of the ensuing free oscillations gives the following results:

$$x_{\max} = 30\,\mathrm{mm}, \qquad \dot{x}_{\max} = 60\,\mathrm{mm\,s^{-1}}.$$

Compute:

1. the total mass $m = m_1 + m_2$
2. the mass m_2 of the impacted body,
3. the circular frequency of the insuing motion,
4. the spring stiffness k.

2 Vibration Isolation — Numerical Integration

A rotating machine, its mass $M = 35\,000\,\mathrm{kg}$, is rigidly connected to the floor.

Due to unbalances, during steady-state regime the machine is subjected to a harmonic force $p(t) = 1\,\mathrm{kN}\,\sin(2\pi\,5\,\mathrm{Hz}\,t)$.

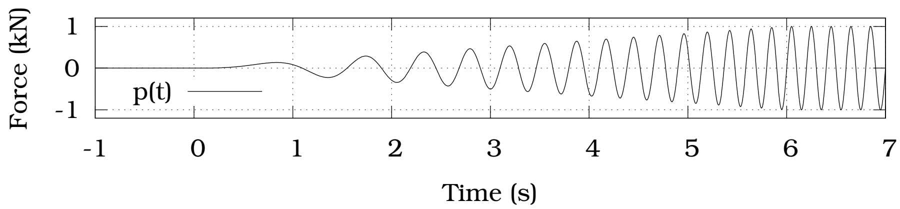
2.1 Vibration Isolation

Considering the floor fixed, design an appropriate suspension system such that the steady-state transmitted force is reduced to $300\,\mathrm{N}$.

2.2 Numerical Integration

When the machine is turned on, its full velocity is reached in $6\,\mathrm{s}$. The angular velocity and the unbalanced load vary linearly, from 0 to their respective maximum values, i.e.,

$$p(t) = \begin{cases} 1\,\mathrm{kN}\,\frac{t}{6\,\mathrm{s}}\,\sin\left(2\pi\,2.5\,\mathrm{Hz}\,\frac{t^2}{6\,\mathrm{s}}\right) & 0\,\mathrm{s} \le t \le 6\,\mathrm{s}, \\ 1\,\mathrm{kN}\,\sin(2\pi\,5\,\mathrm{Hz}\,t) & 6\,\mathrm{s} \le t. \end{cases}$$



Using the stiffness computed in the previous step, find the maximum absolute value of the displacement using either the constant or the linear acceleration method and plot the response in the interval $0\,\mathrm{s} \le t \le 10\,\mathrm{s}$.

3 Estimation of damping ratio

You want to determine the mass m, the stiffness k and the damping ratio ζ of a one storey building that can be modeled as a single degree of freedom system.

A series of 4 dynamical test is performed, loading the building with a vibrodyne and measuring the amplitude ρ and the phase difference θ of the steady state motion (note that the measures of ρ and θ are affected by a random measurement error).

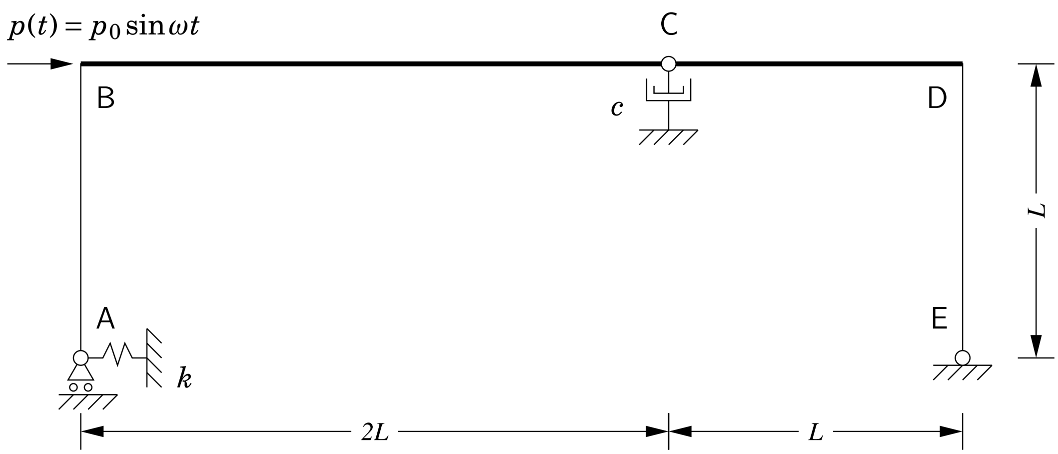
In each test the load amplitude is $p_0 = 600\,\mathrm{N}$, while the excitation frequencies ω_n (with $n = 1,\dots,4$) are different.

The relevant data is summarized in the following table

n	$\omega_n(\mathrm{rad\,s^{-1}})$	$\rho_n(\mu\mathrm{m})$	$\theta_n(\mathrm{deg})$
1	40	12.39062	7.58258
2	50	41.09556	33.33505
3	60	18.07490	163.21210
4	70	7.11246	171.69968

Give your best estimate of m, ζ and k.

4 Generalised Coordinates (rigid bodies)



The articulated system in figure, composed by

- two rigid bars, (1) ABC and (2) CDE,
- three fixed constraints, (1) a horizontal roller in A, (2) an internal hinge in C and (3) a hinge in E,
- two deformable constraints, (1) a horizontal spring in A, its stiffness = k and (2) a vertical dashpot in C, its damping coefficient = c,

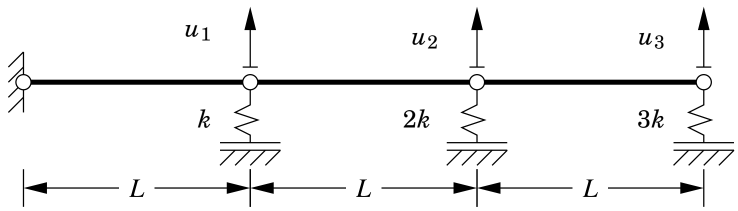
is excited by a horizontal harmonic force applied in B, $p(t) = p_0 \sin \omega t$.

The vertical parts of the two bars, AB and ED, are massless while both the horizontal parts, BC and CD, have a constant unit mass $\overline{m}$, with $\overline{m} L = m$.

Using u_{A} (the horizontal displacement of A) as the generalised coordinate

1. compute the generalised parameters m^*, c^* and k^*,
2. compute the generalised loading $p^*(t)$ and
3. write the equation of dynamic equilibrium.

5 Rayleigh quotient



The undamped 3 *DOF* system in figure is composed of 3 identical rigid bars, their masses $m_i = m$, and three vertical springs, their stiffnesses as detailed in figure. Starting with a trial shape $\boldsymbol{\phi} = \begin{Bmatrix}1 & 1 & 1\end{Bmatrix}^T$ so that $u_1 = u_2 = u_3 = Z_0 \sin \omega t$, give the successive Rayleigh estimates of (squared) free vibration circular frequency R_{00}, R_{01} and R_{11}.

Note (1) that the bars have a not negligible rotatory inertia: $J_i = mL^2/12$, that you must take into account and (2) that the free coordinates are not referred to the centres of mass of the bars (hence a non-diagonal mass matrix).

HINT: the nodal inertial forces are $\boldsymbol{f}_\text{I} = \boldsymbol{M}\,\ddot{\boldsymbol{u}}$, the mass matrix's coefficients can be deduced comparing an explicit derivation of the kinetic energy T in terms of the velocities $\dot{u}_i$, the mass m and the inertia J to the matrix expression $T = \frac{1}{2}\dot{\boldsymbol{u}}^T \boldsymbol{M}\,\dot{\boldsymbol{u}} = \frac{1}{2}\left(m_{11}\dot{x}_1^2 + \cdots + (m_{12} + m_{21})\dot{x}_1\dot{x}_2 + \cdots\right)$, where $m_{ij} = m_{ji}$.

6 3 DOF System

With reference to the system of problem 5, using the position $\omega_0^2 = \dfrac{k}{m}$

1. compute the three eigenvalues of the system and the corresponding eigenvectors,
2. normalize the eigenvectors with respect to the mass matrix $\boldsymbol{M}$ (it must be $\boldsymbol{\psi}^T \boldsymbol{M}\,\boldsymbol{\psi} = m$).

Considering that the system is at rest for $t = 0$ and is then loaded by a load vector $\boldsymbol{p}(t)$,

$$\boldsymbol{p}(t) = \frac{kL}{200}\begin{Bmatrix} 0 \\ -1 \\ +1 \end{Bmatrix}\sin(7\omega_0 t),$$

3. find the analytical expression of $u_3 = u_3(t)$, showing your intermediate results and
4. plot u_3 in the interval $0 \le \omega_0 t \le 6$.