Dynamics of structures

Second Home Assignment, due on your oral exam

The day of your oral examination you shall submit to me a printed or handwritten paper with your solutions to these four problems.

Contents

1 Differential support motion

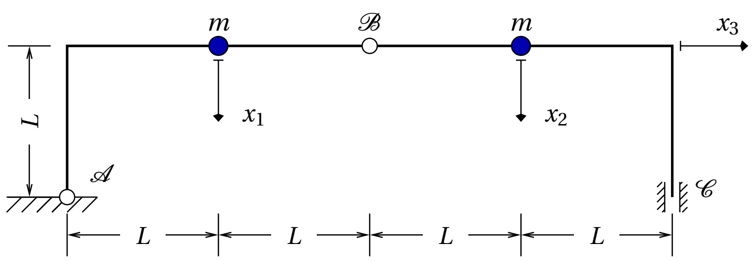


Figure 1: structural model

The system in figure 1 is composed by two uniform elastic beams, hinged in $\mathscr{B}$ and connected to the ground in $\mathscr{A}$ and in $\mathscr{C}$. The restraint in $\mathscr{C}$ permits only the vertical displacement of the beam (no horizontal displacement, no rotation).

The system supports two concentrated masses, $m \gg \bar{m}L$, so that the system can be studied as a 3 DOF dynamical system, using the dynamical degrees of freedom indicated in figure 1.

Disregarding shear and axial deformations, the flexibility matrix for the dynamical degrees of freedom can be easily computed using the *PVD*,

$$\boldsymbol{F} = \frac{L^3}{6EJ}\begin{bmatrix} 26 & 62 & 3 \\ 62 & 162 & 9 \\ 3 & 9 & 2 \end{bmatrix},$$

from which follows the stiffness matrix

$$\boldsymbol{K} = \frac{3EJ}{260L^3}\begin{bmatrix} 243 & -97 & 72 \\ -97 & 43 & -48 \\ 72 & -48 & 368 \end{bmatrix}.$$

Just to be sure, the mass matrix is

$$\boldsymbol{M} = m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

1. Compute the eigenvalues in terms of $\omega_0^2 = {}^{EJ}/_{mL^3}$.
2. Compute the normalised eigenvector matrix.

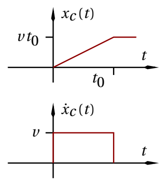
The system is excited by an imposed horizontal displacement applied in $\mathscr{C}$.

3. Compute the 3×1 influence matrix **E**.

   When you permit a horizontal displacement in $\mathscr{C}$, the remaining restraints allow a rigid motion of the system. This affects the way you compute $\boldsymbol{E}$.

The imposed displacement has the following analytical expression,

$$x_{\mathscr{C}}(t) = \begin{cases} 0 & \text{for } t \le 0, \\ vt & \text{for } 0 \le t \le t_0, \\ vt_0 & \text{for } t_0 \le t, \end{cases}$$



where

$$t_0 = \frac{\pi}{\omega_1}, \quad v t_0 = \frac{L}{400}.$$

4. Solve the modal equations of motion, taking into account appropriate initial conditions, in the interval $0 \le t \le t_0$.

   Hints: *a*) the system does not start from rest (initial displacements are zero, initial velocities are not); *b*) the ground acceleration $\ddot{x}_{\mathscr{C}}$ is **always** zero.

5. Give the analytical representation of $x_2(t)$ for $0 \le t \le t_0$.
6. Give a graphical representation of $x_2(t)$ for $0 \le t \le t_0$.

Optional.

7. Solve the modal equation of motion, taking into account appropriate initial conditions, for $t_0 \le t$.

   Hint: as well as there is a discontinuity in the ground velocity at $t = 0$, there is an opposite discontinuity at $t = t_0$, whose effects are opposite to the effects of the first discontinuity.

8. Give the analytical representation of $x_2(t)$ for $t_0 \le t \le 2t_0$.
9. Give a graphical representation of $x_2(t)$ for $t_0 \le t \le 2t_0$.

2 Continuous system

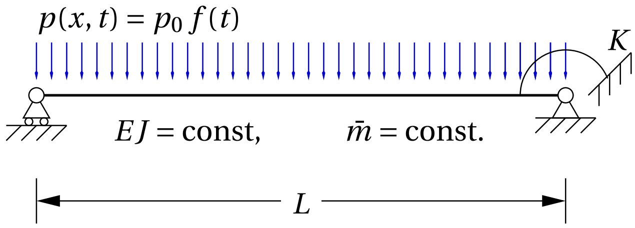


Figure 2: elastically supported beam

The uniform beam in figure 2 is supported by a roller at the left and it is pinned at the right, where its rotation is contrasted by a flexural spring of stiffness $K = kEJ/L$, with $0 \le k < \infty$.

> Note that for $k = 0$ the beam is a simply supported beam, hence $\beta_n L = n\pi$, while for $k \mapsto \infty$ the beam is a clamped-supported beam, hence $\beta_n L \approx \frac{4n+1}{4}\pi$.
>
> This means that you can bracket your results for intermediate values of k.

The beam is loaded by a distributed load $p(x,t) = p_0 f(t)$.

1. With $k = 2$, find β_n, ω_n, $\phi_n(x)$, m_n, and $p_n(t)$ for $n = 1,2,3$.
2. Always with $k = 2$, plot $\phi_n(x)$ vs $\frac{x}{L}$ for $n = 1,2,3$.
3. For $10^{-3} \le k \le 10^3$, plot $\beta_n(k)$ with $n = 1,2,3$, using a logarithmic x-axis.

Optional.

4. For $10^{-3} \le k \le 10^3$, plot $m_n(k)$ and $p_n(k)$ with $n = 1,2,3$, using a logarithmic x-axis..

3 Matrix iteration

Consider a shear type building with N stories, $N = 12$, numbered from the base to the top.

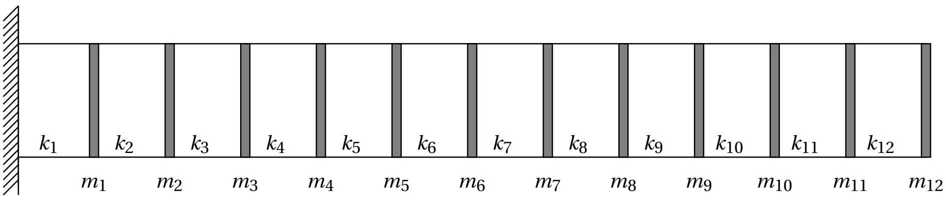


Figure 3: the shear type building of problem 3, rotated clockwise by 90^o.

The storey masses m_n are given by $m_n = m \times (31 - n)$ $[n = 1,\ldots,N]$ where m is a unit mass.

The storey stiffnesses k_n are given by $k_n = k \times (200 - 5n - n^2)$ $[n = 1,\ldots,N]$ where k is a unit stiffness.

Note that the storey stiffness k_n is different from the diagonal term of the stiffness matrix k_{nn}, $k_{nn} = k_n + k_{n+1}$.

1. Using the following initial Ritz base $\mathbf{\Phi}_0$,

$$\mathbf{\Phi}_0^T = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 1 & 2 & 3 & 5 & 5 & 4 & 3 & 1 & -1 & -3 & -6 & -9 \\ 1 & 3 & 3 & 1 & -1 & -3 & -3 & -1 & 1 & 3 & 6 & 10 \end{bmatrix}$$

   and the subspace iteration procedure give an estimate of ω_1^2, ω_2^2 and $\mathbf{\Psi}_1$, $\mathbf{\Psi}_2$, the first two eigenvalues and eigenvectors of the structure, using at least two iterations.

4 Inelastic design

A SDOF system has a natural frequency of vibration $f_n = 2.0\,\text{Hz}$ and a yield strength $f_y = 0.40w$, where w is the system's weight.

The design maximum ground acceleration is 0.4g, the amplification factor for spectral accelerations (elastic) is $\alpha_A = 2.2$ and for the spectral region of concern it is $A = \alpha_A \ddot{x}_{g0}$.

1. Find the system's required ductility μ and its peak displacement x_m .

2. What can be done to reduce by 5% the required ductility?