Dynamics of Structures 2010-2011

the *In the Summertime* home assignment

Contents

Instructions

This assignment is due on Thursday 8th of September. You must check in your assignment by email (`giacomo.boffi@polimi.it`), in the form of a PDF[1] [2] attachment with your solutions to the problems.

For each of the *six problems* copy the text of the problem, summarize the procedure you'll be using, detail all relevant steps including part of intermediate numerical results as you see fit, explicitly give the required answers. Please do not include program sources and spreadsheets directly in your paper, but submit them as attached files.

Points are ≈ 10 15 15 35 35 30 for exact results and clean developments. Minimum required is 85 points. Correct answers to all the optional questions give an extra 15 points.

You can discuss the assignments with your colleagues, but your paper **must** be strictly the result of your individual effort.

[1] By PDF, I mean something produced by LaTeX or Office or similar.

[2] Please check that all needed fonts are included in the file before sending. In doubt, http://en.allexperts.com/q/Microsoft-Word-1058/2009/8/embedding-pdf-file.htm.

1 Dynamical Testing

A simple structure, which can be modeled as a single degree of freedom system, is subjected to testing to measure its dynamical characteristics.

First, it is loaded with a static force $F = 20\,\mathrm{kN}$ and the static displacement is measured: $u_0 = 18\,\mathrm{mm}$, then the force is suddenly released, the structure oscillates freely and after 10 cycles, corresponding to 6 s after the force release, the measured maximum displacement is $u_{10} = 6\,\mathrm{mm}$.

What are the parameters of the SDOF system?

2 Vibration Isolation

A testing apparatus is to be installed in an industrial building.

Due to the presence of a large rotating machinery, the building floor is subjected to a vertical harmonic displacement with a frequency $f_0 = 30\,\mathrm{Hz}$ and an amplitude $\Delta = 60\,\mu\mathrm{m}$.

To guarantee a correct operation, the maximum vertical displacement of the apparatus must be less than $10\,\mu\mathrm{m}$.

1. Knowing that the total weight of the apparatus and its support is 6 kN, design a suitable suspension system.
2. What is the vertical displacement when a technician, his/her weight 1 kN, steps on the support?
3. Is it possible to increase the static stiffness of the suspension system without incrementing the dynamical displacements?

3 Rayleigh Quotient

A tower-like structure is composed by a vertical cantilever beam, its height $H = 80\,\mathrm{m}$, that supports on the top a horizontal platform .

The beam is in reinforced concrete (Young modulus $E = 30\,\mathrm{GPa}$, mass density $\rho = 2500\,\mathrm{kg\,m^{-3}}$), its constant section is annular with a thickness $t = 0.30\,\mathrm{m}$ and an unknown external radius R_{ext}; the platform is circular, its radius is $R_{\mathrm{p}} = 6.0\,\mathrm{m}$, its specific mass is $\gamma = 1400\,\mathrm{kg\,m^{-2}}$.

Using the Rayleigh quotient method, considering the platform as a point mass, determine R_{ext} so that the estimated natural period of vibration of the structure is equal to 2.0 s.

Hint

For an annular section you can write $A = 2\pi R_{\mathrm{m}} t$ and $J \approx \pi R_{\mathrm{m}}^3 t$, where R_{m} is the mean radius, $R_{\mathrm{m}} = R_{\mathrm{ext}} - \frac{t}{2}$. Using the above expressions for A and J leads to a simple equation in R_{m}.

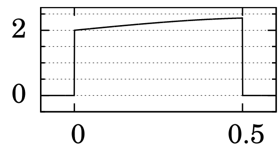
Optional

Using the value of R_{ext} previously determined, compute the period of vibration of the first mode of the structure according to the theory of free vibrations of continuous beams.

4 Numerical Integration

A single degree of freedom system, with a mass $m = 1500\,\text{kg}$, a stiffness $k = 60\,\text{kN}\,\text{m}^{-1}$ and a damping ratio $\zeta = 0.03$ is at rest when it is subjected to an external force $p(t)$:

$$p(t) = \begin{cases} 2\,\text{kN}\left(1 + \frac{1}{2}at - \frac{1}{2}(at)^3\right) & \text{for } 0.0 \le t \le 0.50\text{s}, \\ 0.0 & \text{otherwise,} \end{cases}$$



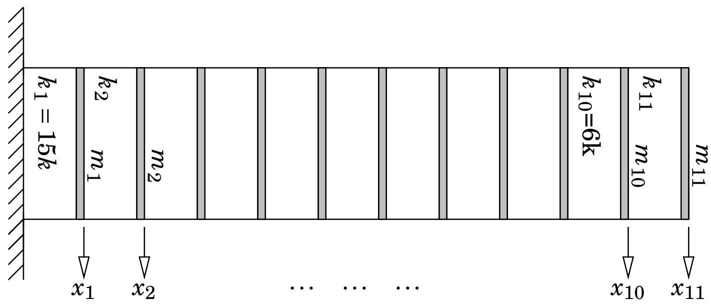
where $a = 1\,\text{s}^{-1}$.

1. Find the exact response in the time interval $0 \le t \le 3\,\text{s}$, using superposition of the general integral, $x(t) = \exp(-\zeta\omega t)[A\sin\omega_D t + B\cos\omega_D t]$, and appropriate particular solutions.
2. In the same interval, integrate the equation of motion numerically, using the algorithm of linear acceleration with a time step $h = 0.02\,\text{s}$.
3. Plot your results (both the exact response and the numerical solution) in a meaningful manner .

Optional

Repeat the numerical integration assuming an elasto-plastic spring with a yield strength $f_y = 3.0\,\text{kN}$ and plot your results.

5 Rayleigh-Ritz & Subspace Iteration



The structure above can be analyzed as a shear type building. The storey masses vary linearly from $m_1 = 40m$ to $m_{11} = 30m$, also the inter-storey stiffnesses vary linearly from $k_1 = 15k$ to $k_{11} = 5k$.

Note that the inter-storey stiffnesses k_i are not the diagonal terms $k_{i,i}$ of the stiffness matrix, e.g., $k_{4,4} = k_4 + k_5 = 12k + 11k = 23k$.

1. Starting with the Ritz base, derived by considering n^0, n^1 and n^2,

$$\Phi_0^T = \begin{bmatrix} 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 0.10 & 0.20 & 0.30 & 0.40 & 0.50 & 0.60 & 0.70 & 0.80 & 0.90 & 1.00 & 1.10 \\ 0.01 & 0.04 & 0.09 & 0.16 & 0.25 & 0.36 & 0.49 & 0.64 & 0.81 & 1.00 & 1.21 \end{bmatrix}$$

   a: solve the reduced eigenproblem in Ritz coordinates (write the reduced matrices, the eigenvalues and the eigenvectors in Ritz coordinates), **b:** find the first three eigenvectors in structural coordinates, **c:** using subspace iteration derive a new Ritz base Φ_1, **d:** using the new Ritz base, solve the reduced eigenproblem in Ritz coordinates (write the reduced matrices, the eigenvalues and the eigenvectors in Ritz coordinates), **e:** find the first three eigenvectors in structural coordinates.

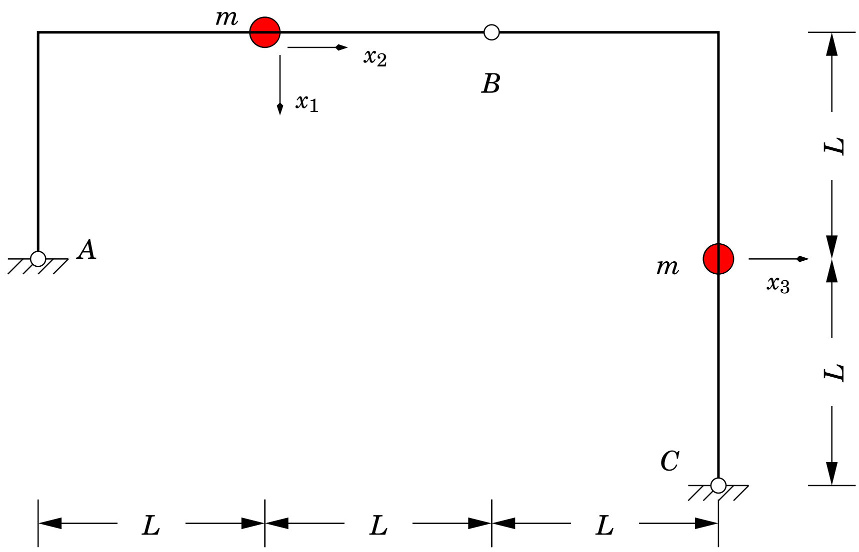
2. Starting with the Ritz base, derived by an empirical knowledge of mode shapes,

$$\Phi_0^T = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & 3 & 3 & 2 & 1 & 0 & -1 & -2 & -3 & -4 \\ 1 & 2 & 1 & 0 & -1 & -2 & -2 & -1 & 0 & 1 & 3 \end{bmatrix}$$

   a: solve the reduced eigenproblem in Ritz coordinates (write the reduced matrices, the eigenvalues and the eigenvectors in Ritz coordinates), **b:** find the first three eigenvectors in structural coordinates, **c:** using subspace iteration derive a new Ritz base Φ_1, **d:** using the new Ritz base, solve the reduced eigenproblem in Ritz coordinates (write the reduced matrices, the eigenvalues and the eigenvectors in Ritz coordinates), **e:** find the first three eigenvectors in structural coordinates.

3. Discuss the two set of results and what could be a good choice of the initial Ritz base.

6 3 *DOF* System



A structure, composed of two uniform beams with the same sectional properties and the same material properties, sustains two equal point masses.

The beam masses are negligible with respect to the suspended masses, the flexibility matrix

$$\boldsymbol{F} = \frac{L^3}{30EJ} \begin{bmatrix} 8 & 6 & 6 \\ 6 & 24 & 15 \\ 6 & 15 & 14 \end{bmatrix}$$

has been computed neglecting the axial and shear deformations of the beams.

1. Write the structural matrices $\boldsymbol{K}$ and $\boldsymbol{M}$, find the eigenvalues solving the equation of frequencies, find the eigenvectors of the system.

2. Examine the behaviour of the system when its supports are subjected to a horizontal earthquake excitation, $\ddot{u}_g = \ddot{u}_g(t)$.

Optional

Repeat point 2 above, when only point C is subjected to a given horizontal motion, $u_C(t)$.