Dynamics of Structures 2011-12

Extra Home Assignment, due by February 5 2013

Contents

Instructions

This homework (HW) is due by Tuesday, February 5 2013, in substitution of HW 1 or 3 AND in substitution of HW 2. **This is the only HW you need to submit to enter the February exams.**

The maximum scores for each one of the seven problems are respectively (and approximately) 12, 6, 6, 16, 18, 20, 22 (100 points total).

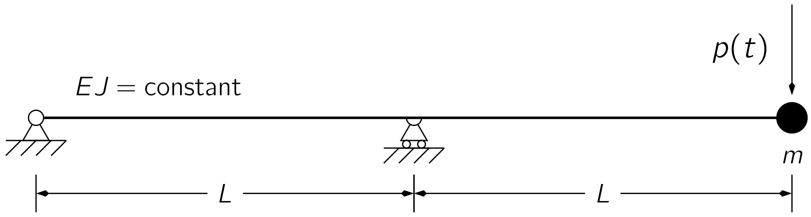
For each one of the seven problems (a) copy the text of the problem, (b) summarize the procedure you'll be using, (c) write down all the relevant steps (showing parts of the intermediate numerical results as you see fit), (d) clearly state the required answers.

Please submit your homework by email (giacomo.boffi@polimi.it) by Tuesday 5 February CET as a PDF[1] attachment. If you consider it necessary, you can submit spreadsheets, program sources etc as *separate* attachments in the same email.

While you can (and should) discuss the problems with your colleagues, your paper **must** be strictly the result of your individual effort or it will be completely disregarded.

[1] Please check that all the needed fonts are included in the PDF file before sending. If you use Word, http://www.bc.edu/content/dam/files/libraries/pdf/embed-fonts.pdf is a good reference.

1 Dynamical Testing



The simply supported beam in the figure above supports a body of negligible dimensions, its mass $m = 10\,000\,\mathrm{kg}$. The beam mass being very small with respect to the body mass, the system can be modeled as a single degree of freedom system.

The body is at rest when it is subjected to an impulsive loading

$$p(t) = p_0 \begin{cases} \cos\dfrac{\pi\, t}{2t_0} & 0 \le t \le t_0, \\ 0 & \text{otherwise,} \end{cases}$$

where $p_0 = 2.0\,\mathrm{kN}$ and $t_0 = 0.1\,\mathrm{s}$.

The maximum displacement δ_{max} of the supported body occurs *after* the load phase, with $\delta_{\mathrm{max}} = 4.0\,\mathrm{mm}$.

What is the flexural stiffness EJ of the beam if $L = 6\,\mathrm{m}$?

Hint

For an impulsive load, you can write

$$\delta_{\mathrm{max}} \approx \frac{1}{m\,\omega_n}\int_0^{t_0} p(\tau)\,d\tau = \frac{1}{\sqrt{m\,k}}\int_0^{t_0} p(\tau)\,d\tau$$

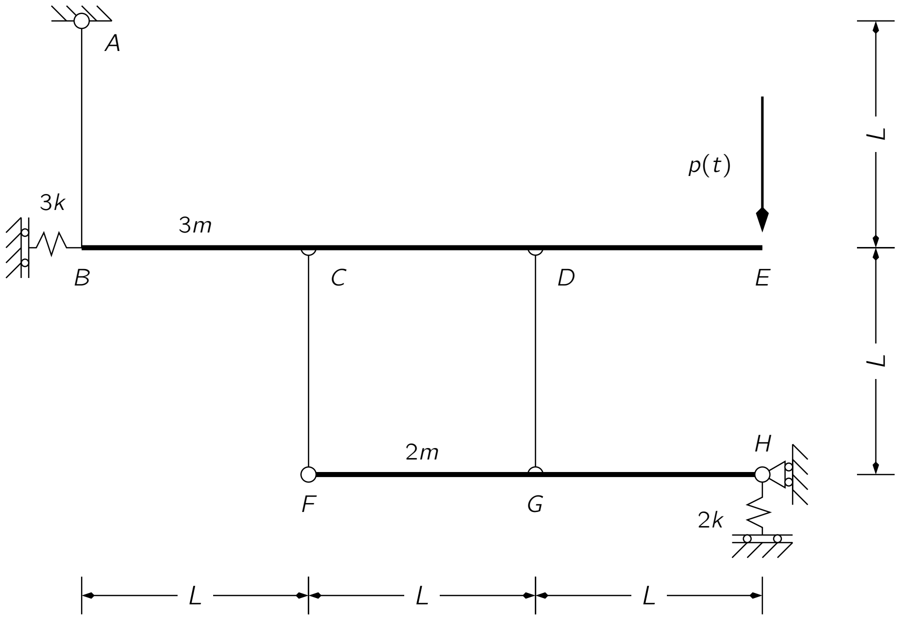
where k, the system stiffness, can be easily computed in terms of EJ and L.

2 Vibration Isolation

A large machine weights 800 kN. When the machine reaches the steady state, due to unbalances in its rotating parts, a harmonic force of 1.6 kN at 20 Hz is transmitted to its rigid foundation.

Design a suspension system, characterized by a damping ratio $\zeta = 0.12$, so that the steady-state transmitted force is reduced to 400 N.

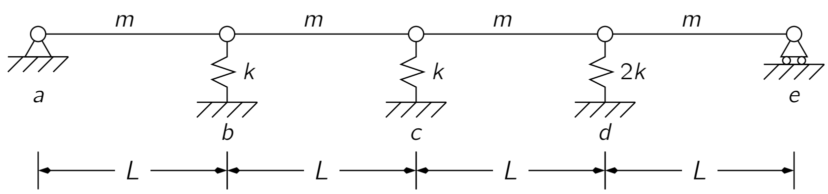
3 Generalized Coordinates



The system in the figure above is composed by 4 rigid rods and two elastic springs and it is a single degree of freedom system.

Using v_E, the vertical displacement of the point E, as the free coordinate write the equation of motion for the SDOF system, knowing that the vertical parts of the rods are massless and that the horizontal parts have a constant unit mass, $\bar{m} = {}^{m}/_{L}$ (i.e., $\overline{BCDE}$ has a total mass $m_{BCDE} = 3m$, etc).

4 Rayleigh Quotient



The system in the figure above is composed by four identical rigid, straight rods, of mass m and length L, and by three elastic springs (NB the right spring is stiffer than the other two).

Using the internal hinges vertical displacements as the degrees of freedom, find an approximation to the first eigenvalue of the system using the Rayleigh quotient technique and a trial vector $\phi = \begin{Bmatrix} 1 & 1 & 1 \end{Bmatrix}^T$, then find the 01 and 11 refinements of the eigenvalue (remember to take into account the rotatory inertia of the rods).

5 Rayleigh-Ritz Procedure

You can download the mass matrix and the stiffness matrix of a 48 DOF dynamical system using the links in red.

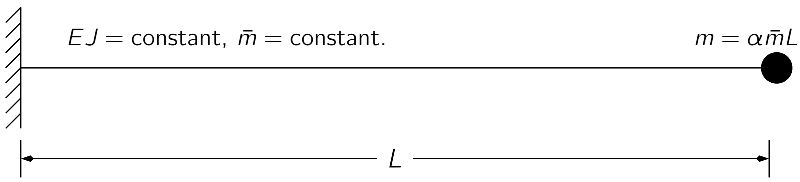
The matrices you're going to download were saved in the so called Matrix Market format (in short, for the cases at hand you have in each non-header row the coordinates i, j and the value $a_{ij} = a_{ji}$ of the non-zero entries in the matrix). A matlab function to read Matrix Market formatted matrices is available starting from the link above.

Note that $\boldsymbol{M}$ is a semi-definite positive matrix, as some of the diagonal elements are equal to zero: the number of Dynamical DOF is 24.

Find the first two eigenvalues/eigenvectors of the system using the Rayleigh-Ritz procedure, with a number of base vectors $m = 4$, stopping the iterations when ${}^{\omega^2_{2,n+1}-\omega^2_{2,n}}/_{\omega^2_{2,n}} \leq 10^{-4}$.

For each iteration, print only the eigenvalues and the reduced matrices, where the new Ritz base is computed from the *normalized* eigenvectors.

6 Continuous System



An uniform, elastic beam, its flexural stiffness EJ = constant and its unit mass $\bar{m}$ = constant, is clamped at one side and supports a body, its mass $m = \alpha \bar{m} L$, at the other side.

❶ Write the boundary conditions. ❷ With $\alpha = 2$, for the first three modes, give (a) the frequencies of vibration, (b) the normalized eigenfunctions' analytical expressions (remember to include the lumped mass contribution when normalizing), as well as a plot of them, (c) the values of the modal loads for a spatially uniform load $p(x,t) = p_0 f(t)$. ❸ For $10^{-2} \leq \alpha \leq 10^2$ plot the first three eigenvalues as functions of α, $\omega_n^2 = \omega_n^2(\alpha)$, $n = 1, 2, 3$, against a logarithmic α axis.

7 Support Motion

The system of exercise 4 is subjected to a forced displacement at the base of central spring,

$$v_c(t) = \frac{L}{1000} \begin{cases} 0 & t \leq 0 \\ 1 - \cos(\pi \, {}^{t}/_{t_0}) & 0 \leq t \leq t_0 \\ 2 & t_0 \leq t \end{cases}$$

with $\omega_0 \, t_0 = \sqrt{{}^{k}/_{m}} \, t_0 = \pi/2$.

❶ Compute $\boldsymbol{M}$ and $\boldsymbol{K}$. ❷ Compute the eigenvalues (if you use matlab sort the eigenvalues) and the normalized eigenvectors. ❸ Compute $\boldsymbol{E}$, the influence matrix (NB when you remove the support in c you have a mechanism). ❹ Write the modal equations of motion in terms of the adimensional coordinates $\chi_n = 1000\, q_n/L$, in the interval $\omega_0 t = 3\pi$. ❺ Integrate the modal equations and plot the modal responses $\chi_n = \chi_n(\omega_0 t)$ in the same interval. ❻ Plot the nodal displacements, using the same normalization factor. ❼ Compute numerically the response, in the same interval, using the constant acceleration algorithm with a time step $h = {}^{t_0}/_{40}$. ❽ Is the time step appropriate? ❾ Plot the numerically computed nodal displacements.