Dynamics of Structures 2011-2012

1st home assignment due on Tuesday 2012-05-22

Contents

Instructions

For each of the *six problems* copy the text of the problem, summarize the procedure you'll be using, detail all relevant steps including part of intermediate numerical results as you see fit, *clearly state* the required answers.

The max scores for each exercise are respectively (and approximately) 8, 11, 20, 16, 20 and 25, for a total of 100 points for a *perfect* submission.

Please submit your homework by email before 14:30: email submissions *must comprise a PDF*[1] *attachment* with your solutions and, optionally, the spreadsheets and/or the sources of the programs that you used for computing your solutions.

Alternatively to an email submission, if you so prefer you can hand in your homework directly to me before the class. Handed in submissions must be printed or *nicely* handwritten.

[1]Please check that all the needed fonts are included in the PDF file before sending. If you use Word, http://www.bc.edu/content/dam/files/libraries/pdf/embed-fonts.pdf is a good reference.

1 Dynamical Testing

A simple structure, which can be modeled as a single degree of freedom system, is subjected to testing to measure its dynamical characteristics:

1. the structure is loaded with a static force $F = 12.0\,\mathrm{kN}$ and the static displacement is measured: $u_0 = 2.0\,\mathrm{mm}$,

2. the force is then suddenly released, the structure oscillates freely and after 12 cycles, corresponding to $3.0\,\mathrm{s}$ after the force release, the measured maximum displacement is $u_{12} = 0.6\,\mathrm{mm}$.

What are the values of m, ζ and k?

2 Vibration Isolation

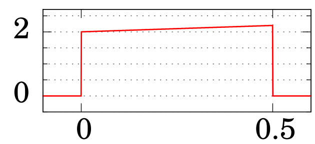
A rotating machine is characterized by its mass $m = 192\,000\,\mathrm{kg}$, its working frequency $f_\mathrm{w} = 100\,\mathrm{Hz}$ and the value of the unbalanced load it exerts on its supports, $f_\mathrm{w} = 4800\,\mathrm{N}$.

Design a suspension system for the machine (i.e., give the values of k and c, the stiffness and the damping constant) knowing that (1) it is necessary to reduce the transmitted force to $400\,\mathrm{N}$, (2) to reduce the vibration amplitude during transients the suspension must have a viscous damping ratio of 6%.

3 Numerical Integration

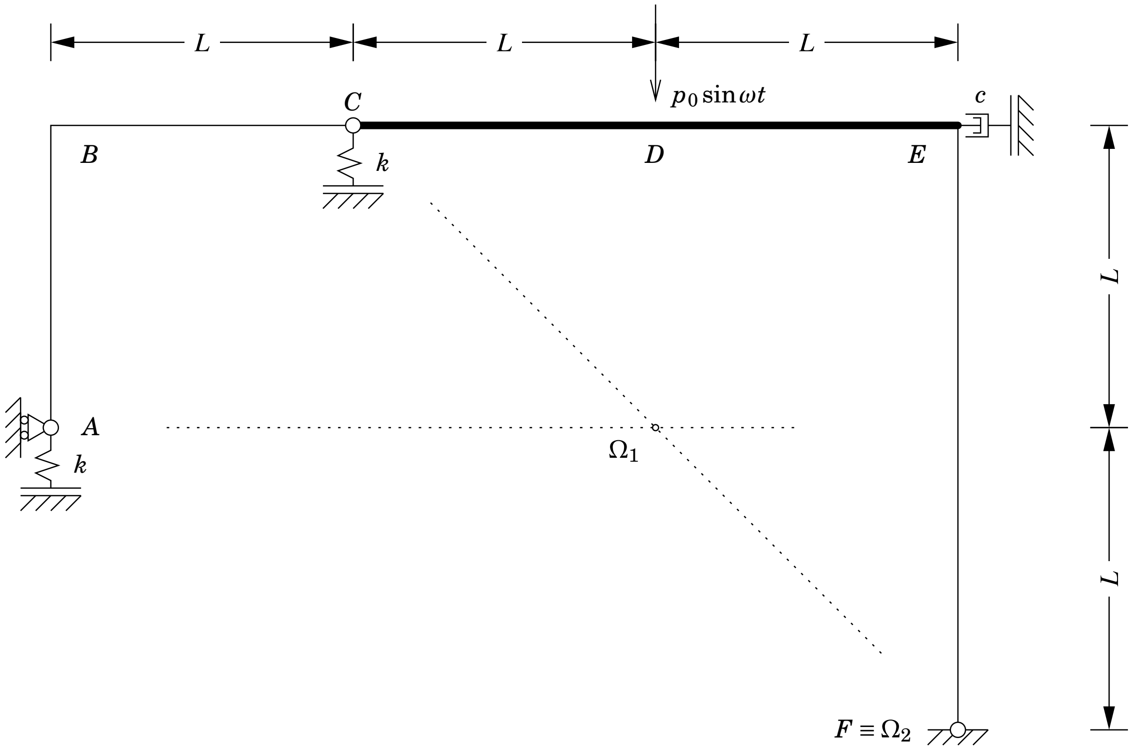
A single degree of freedom system, with a mass $m = 1200\,\mathrm{kg}$, a stiffness $k = 50\,\mathrm{kN\,m^{-1}}$ and a damping ratio $\zeta = 0.05$ is at rest when it is subjected to an external force $p(t)$:

$$p(t) = \begin{cases} 2\,\mathrm{kN}\left(0.95 + \frac{1}{10}\frac{t}{t_0}\right) & \text{for } 0.0 \le t \le t_0 = 0.50\,\mathrm{s}, \\ 0.0 & \text{otherwise.} \end{cases}$$



(1) Write the exact response for $0 \le t \le t_0$ and $t_0 \le t \le 2\,\mathrm{s}$ using superposition of the general integral and appropriate particular integrals. (2) Integrate the equation of motion numerically, using the algorithm of linear acceleration with a time step $h = 0.02\,\mathrm{s}$ for $0 \le t \le 2\,\mathrm{s}$. (3) Plot your results (both the exact response and the numerical solution) in a meaningful manner. (4) [OPTIONAL] Repeat the numerical integration assuming an elasto-plastic spring with a yield strength $f_y = 3.2\,\mathrm{kN}$ and plot your results.

4 Generalized Coordinates (rigid bodies)



The articulated system in figure, composed by

- two rigid bars, (1) ABC and (2) CDEF,
- three fixed constraints, (1) a vertical roller in A, (2) an internal hinge in C and (3) a hinge in F,
- three deformable constraints, (1) a vertical spring in A, its stiffness k, (2) a vertical spring in C, its stiffness k and (3) a horizontal dashpot in E, its damping coefficient c,

is excited by a vertical harmonic force applied in D, $p(t) = p_0 \sin\omega t$.

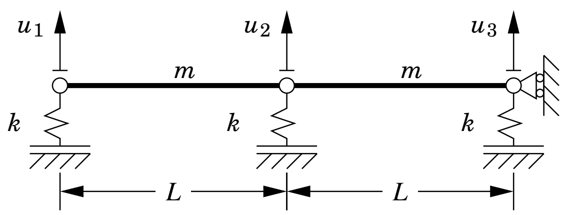
The horizontal part of the bar CDEF has a constant unit mass $\overline{m}$, with $\overline{m}L = m$; all the other parts of the system are massless.

Using v_C (the vertical displacement of C) as the generalized coordinate

1. compute the generalized parameters m^*, c^* and k^*,
2. compute the generalized loading $p^*(t)$ and
3. write the equation of dynamic equilibrium.

5 Rayleigh quotient

The undamped 3 *DOF* system in figure is composed of 2 identical rigid bars, their masses equal to m, and three identical vertical springs, their stiffnesses equal to k. Use the free coordinates indicated in the figure.



Starting with a trial shape $\boldsymbol{\phi} = \begin{Bmatrix}1 & 1 & 1\end{Bmatrix}^T$ (i.e., $u_1 = u_2 = u_3 = Z_0 \sin\omega t$) give the successive Rayleigh estimates R_{00}, R_{01} and R_{11} of ω^2.

HINTS:

- the bars have a not negligible rotational inertia, $J_i = mL^2/12$, that you should take into account,
- the free coordinates are not referred to the centers of mass of the bars, hence the mass matrix is non-diagonal,
- the simplest way to write the inertial forces on the nodes is using the matrix notation, $\boldsymbol{f}_\mathrm{I} = \boldsymbol{M}\ddot{\boldsymbol{u}}$, where the mass matrix's coefficients can be deduced comparing an explicit derivation of the kinetic energy T in terms of $\dot{u}_i$, m and J to the matrix expression $T = \frac{1}{2}\dot{\boldsymbol{u}}^T\boldsymbol{M}\dot{\boldsymbol{u}} = \frac{1}{2}\left(m_{11}\dot{u}_1^2 + \cdots + (m_{12} + m_{21})\dot{u}_1\dot{u}_2 + \cdots\right)$, where $m_{ij} = m_{ji}$.

6 3 DOF System

With reference to the system of problem 5, using the position $\omega_0^2 = \dfrac{k}{m}$

1. compute the three eigenvalues of the system and the corresponding eigenvectors,
2. normalize the eigenvectors with respect to the mass matrix $\boldsymbol{M}$ (it must be $\boldsymbol{\Psi}^T \boldsymbol{M} \boldsymbol{\Psi} = m$).

Considering that the system is at rest for $t = 0$ and is then loaded by a load vector $\boldsymbol{p}(t)$,

$$\boldsymbol{p}(t) = \frac{kL}{500} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \sin(2\omega_0 t),$$

3. write the three *modal* equations of motion,
4. integrate the modal equations of motion and write the three equations of modal displacement, $q_i = q_i(t)$,
5. find the analytical expression of $u_3 = u_3(t)$, showing your intermediate results and
6. plot u_3 in the interval $0 \le \omega_0 t \le 10$.