Dynamics of Structures 2011-2012

Second Home Assignment, due on your oral exam

The day of your oral examination you shall submit to me a printed or handwritten paper with your solutions to the following three problems.

1 Differential support motion

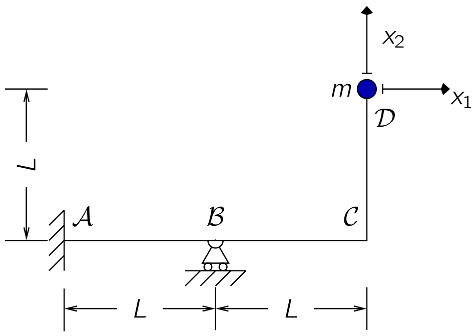

Figure 1: structural model

The system in figure 1 is composed by a uniform elastic beam, clamped in $\mathcal{A}$ and simply supported (roller) in $\mathcal{B}$, that in $\mathcal{D}$ supports a concentrated mass m; as the supported mass is much larger than the mass of the beam ($m \gg 3\bar{m}L$) the system can be studied as a 2 DOF dynamical system, using the dynamical degrees of freedom indicated in figure.

Disregarding shear and axial deformations, the flexibility matrix for the dynamical degrees of freedom can be easily computed using the *PVD*,

$$\boldsymbol{F} = \frac{L^3}{12EJ}\begin{bmatrix}19 & -9\\ -9 & 7\end{bmatrix},$$

from which follows the stiffness matrix

$$\boldsymbol{K} = \frac{3EJ}{13L^3}\begin{bmatrix}7 & 9\\ 9 & 19\end{bmatrix}.$$

1. Compute the eigenvalues in terms of $\omega_0^2 = {}^{EJ}/_{mL^3}$.
2. Compute the normalised eigenvector matrix.

The system is excited by an imposed vertical displacement applied in $\mathcal{B}$.

3. Compute the 2×1 influence matrix $\mathbf{E}$.

The imposed displacement has the following analytical expression,

$$x_{\mathcal{B}}(t) = \begin{cases} 0 & \text{for } t \le 0, \\ \frac{L}{1000}\left(3\left(\frac{t}{t_0}\right)^2 - 2\left(\frac{t}{t_0}\right)^3\right) & \text{for } 0 \le t \le t_0, \\ \frac{L}{1000} & \text{for } t_0 \le t, \end{cases}$$

with $t_0 = \frac{0.1}{\omega_1}$.

4. Solve the modal equations of motion, starting from rest conditions, in the interval $0 \le t \le 50t_0$ and plot your results.

5. Give the analytical representation of $x_1(t)$ for $0 \le t \le 50t_0$ and plot your results.

6. *Optional* Give the analytical representation of the reaction of the roller $R_{\mathcal{B}}(t)$ for $0 \le t \le 50t_0$ and plot your results.

7. *Optional* Discuss the response for $t_0 \mapsto 0$.

2 Continuous system

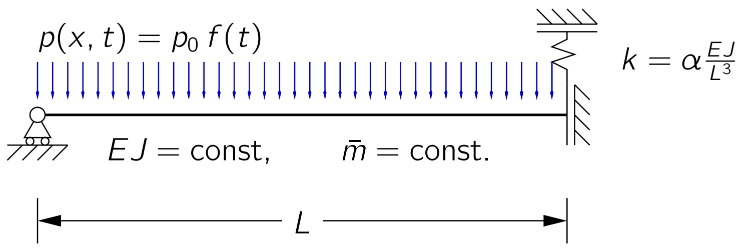

Figure 2: elastically supported beam

The uniform beam in figure 2 is supported by a roller at the left and it is constrained by a double pendulum at the right, where its deflection is contrasted by a spring of stiffness $k = \alpha\, EJ/L^3$, with $0 \le \alpha < \infty$. The beam is loaded by a distributed load $p(x,t) = p_0\, f(t)$.

1. With $\alpha = 10$, find β_n, ω_n, $\phi_n(x)$ (normalize the eigenfunctions so that $m_n = 1$) and $p_n(t)$ for $n = 1, 2, 3$.

2. Always with $\alpha = 10$, plot $\phi_n(x)$ vs $\frac{x}{L}$ for $n = 1, 2, 3$.

3. For $10^{-3} \le \alpha \le 10^3$, plot $\beta_n(\alpha)$ with $n = 1, 2, 3$, using a logarithmic x-axis.

   Note that for $\alpha = 0$ the beam is equivalent to a simply supported beam of length $2L$ for which the even modes are impossible, hence $2\beta_n L = (2n-1)\pi$, while for $\alpha \mapsto \infty$ the beam is a clamped-supported beam, hence $\beta_n L \approx \frac{4n+1}{4}\pi$.

3 Matrix iteration

Consider a shear type building with N stories, $N = 11$, numbered from the base to the top.

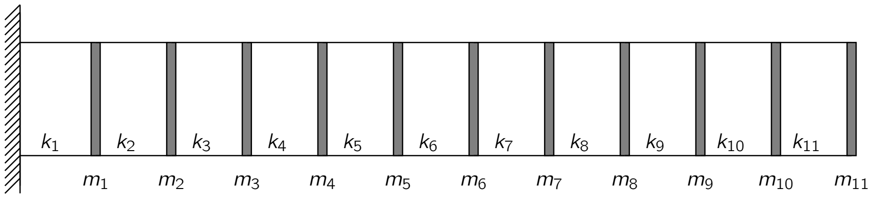

Figure 3: the shear type building of problem 3, rotated clockwise by 90°.

The storey masses m_n are given by $m_n = m \times (21 - n)$ $[n = 1, \dots, N]$ where m is a unit mass.

The storey stiffnesses k_n are given by $k_n = k \times (204 - 3n - n^2)$ $[n = 1, \dots, N]$ where k is a unit stiffness.

> Note that the storey stiffness k_n is different from the diagonal term of the stiffness matrix k_{nn}, $k_{nn} = k_n + k_{n+1}$.

1. Using the following initial Ritz base $\mathbf{\Phi}_0$,

$$\mathbf{\Phi}_0^T = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & 3 & 5 & 5 & 4 & 3 & 1 & -1 & -3 & -6 \\ 1 & 3 & 3 & 1 & -1 & -3 & -3 & -1 & 1 & 3 & 6 \end{bmatrix}$$

   and the subspace iteration procedure give an estimate of ω_1^2, ω_2^2 and $\boldsymbol{\psi}_1$, $\boldsymbol{\psi}_2$, the first two eigenvalues and eigenvectors of the structure, using at least two iterations and detailing the intermediate steps.

   > To check the results of the subspace iteration procedure you can compute the same eigenvalues and eigenvectors using `matlab` procedure `eig` or the analogous procedure that is offered in your programming language of choice.

2. Using the previous results, recalling that the response can be expressed in terms of the modal response and a static correction

$$\boldsymbol{x} = \boldsymbol{\psi}_1 q_1(t) + \boldsymbol{\psi}_2 q_2(t) + \left(\boldsymbol{K}^{-1} - \boldsymbol{F}_1 - \boldsymbol{F}_2\right) \boldsymbol{r} f(t),$$

   give the static correction term $\left(\boldsymbol{K}^{-1} - \boldsymbol{F}_1 - \boldsymbol{F}_2\right) \boldsymbol{r}$ for a load shape whose components are given by $r_n = -p_0 \times (-1)^n$, detailing the intermediate results.

   > Please note that I'm not interested in the modal response.