Dynamics of Structures 2011-2012

Summer Assignment, due by Friday, 24 August 2012.

Contents

Instructions

The maximum scores for each one of the six problems are respectively (and approximately) 12, 10, 15, 15, 22 and 26 (100 points total).

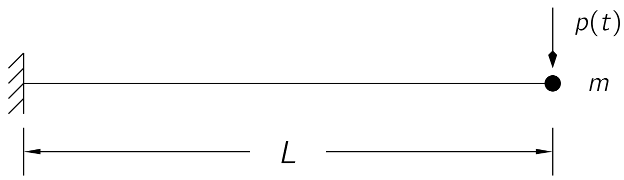
For each one of the six problems **a:** copy the text of the problem, **b:** summarize the procedure you'll be using, **c:** write down all the relevant steps (showing parts of the intermediate numerical results as you see fit), **d:** clearly state the required answers.

Please submit your homework by email (giacomo.boffi@polimi.it) by Friday 24 August as a PDF[1] attachment. If necessary you can submit spreadsheets, program sources etc in *separate* attachments in the same email.

You can (and should) discuss the problems with your colleagues, but your paper **must** be strictly the result of your individual effort.

[1] Please check that all the needed fonts are included in the PDF file before sending. If you use Word, http://www.bc.edu/content/dam/files/libraries/pdf/embed-fonts.pdf is a good reference.

1 Dynamical Testing



A simple structure, which can be modeled as a single degree of freedom system, consists of a uniform cantilever beam of lenght $L = 10\,\mathrm{m}$ that supports a body of negligible dimensions, whose mass $m = 10\,000\,\mathrm{kg}$ is much larger than the beam mass.

The body is at rest when it is subjected to an impulsive loading

$$p(t) = \begin{cases} p_0 \sin(\pi t/t_0) & 0 \le t \le t_0 \\ 0 & \text{otherwise} \end{cases}$$

with $p_0 = 3\,\mathrm{kN}$ and $t_0 = 0.2\,\mathrm{s}$.

The maximum tip deflection, that occurs *after* the end of the loading, in the free vibration phase of the response, is $\delta_{\mathrm{max}} = 7.5\,\mathrm{mm}$.

What is the flexural stiffness EJ of the beam?

Hint

If we denote with T_n the natural period of vibration of the system, it is $T_\mathrm{n} = T_\mathrm{n}(m, EJ)$.

For a half-sine excitation, with $\beta = \dfrac{T_\mathrm{n}}{2t_0}$, if the maximum response occurs in the free vibration phase it is

$$\delta_{\mathrm{max}} = \frac{p_0}{k}\,\frac{2\beta}{\beta^2 - 1}\cos\frac{\pi}{2\beta}.$$

2 Vibration Isolation

An industrial machine has a mass $m_0 = 4200\,\mathrm{kg}$ and transmits to its rigid supports a harmonic force of amplitude $3.6\,\mathrm{kN}$ at $15\,\mathrm{Hz}$ when it reaches the steady state regime.

1. Determine the stiffness k_{susp} of an undamped, elastic suspension system for the machine, so that the amplitude of the s-s transmitted force is no greater than $400\,\mathrm{N}$.

After the installation of the suspension system it is found that the forces transmitted to the supports during the transient are too large,

2. modify the dynamical system parameters (i.e., m, c and k) so that **a:** the transmissibility ratio doesn't increase, **b:** $k = k_{\mathrm{susp}}$, because you want to use the same springs that you have already installed and **c:** the damping ratio is $\xi = 5\,\%$.

3 Numerical Integration

A single degree of freedom system, with mass $m = 500\,\mathrm{kg}$, stiffness $k = 72\,\mathrm{kN\,m^{-1}}$ and damping ratio $\zeta = 0.05$ is at rest when it is subjected to an external force $p(t)$:

$$p(t) = p_0 \exp(-\frac{\omega_n t}{3}),$$

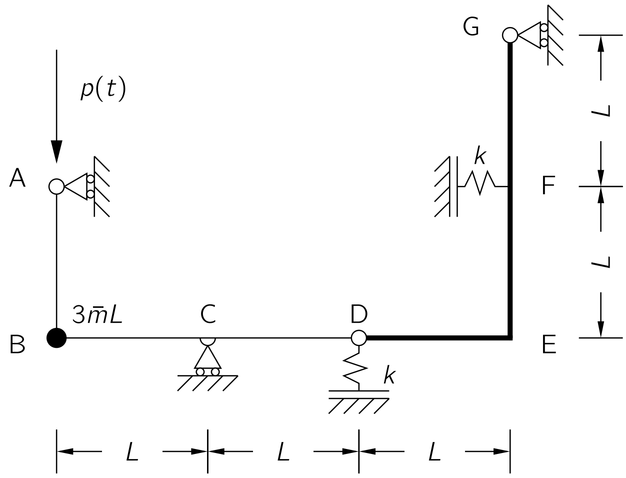
where $p_0 = 5.4\,\mathrm{kN}$ and $\omega_n = 12\,\mathrm{rad\,s^{-1}}$ is the undamped natural frequency of the dynamical system.

1. Find the exact response for $t \geq 0$ using superposition of the general integral and a particular solution.
2. Integrate the equation of motion in the interval $0 \leq t \leq 3\,\mathrm{s}$, using the algorithm of linear acceleration with a time step $h = 0.02\,\mathrm{s}$.
3. Plot your results (exact+numerical) for $0 \leq t \leq 3\,\mathrm{s}$.

Optional

Repeat the numerical integration assuming an elasto-plastic spring with a yield strength $f_y = 6.0\,\mathrm{kN}$ and plot your results.

4 Generalized Coordinates (rigid bodies)



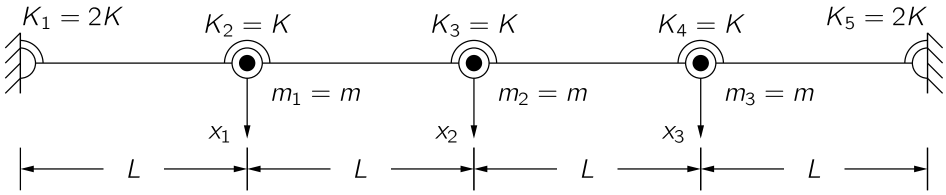
The structure in figure is an articulated SDOF system composed by

- two rigid bars, (1) ABCD and (2) DEFG,
- three fixed external constraints, (1) a vertical roller in A, (2) a horizontal roller in C and (3) a vertical roller in G,
- one internal constraint, a hinge in D
- two deformable external constraints, (1) a vertical spring in D, its stiffness k, (2) a horizontal spring in E, its stiffness k.

The bar DEFG has a unit mass $\bar{m}$, the bar ABCD is massless and supports, in B, a body of negligible dimensions, whose mass is $m = 3\bar{m}L$.
Using v_G (the vertical displacement of G) as the generalized coordinate

1. compute the generalized parameters m^* and k^*,
2. compute the generalized loading $p^\star(t)$ considering a vertical force applied in D, $p(t) = p_0 \sin \omega t$. and
3. write the equation of dynamic equilibrium.

5 Rayleigh Quotient



The system in figure is composed by 4 rigid bars of negligible mass, of equal lenght L, hinged to the system of reference and one to the other, connected to the system of reference and one to the other by 5 discrete flexural springs whose stiffness is indicated in figure (note that the stifnesses are not all the same).

Inside each one of the 3 internal hinges there is a body of negligible dimensions, the mass of each one is indicated in figure. The dynamical degrees of freedom are indicated in figure, too.

Starting with a trial shape $\boldsymbol{\phi} = \begin{Bmatrix} 1 & 1 & 1 \end{Bmatrix}^T$ (i.e., $x_1 = x_2 = x_3 = Z_0 \sin \omega t$) give the successive Rayleigh estimates R_{00}, R_{01} and R_{11} of ω^2, the natural frequency of vibration of the system.

Hint

The deformation energy V can be expressed in terms of the *relative* rotations across the springs and also in terms of the stiffness matrix $\boldsymbol{K}$ coefficients k_{ij} and the nodal displacements:

$$V = \frac{1}{2} \sum_{n=1}^{5} K_n \theta_n^2, \qquad V = \frac{1}{2} \sum_i \sum_j k_{ij} x_i x_j.$$

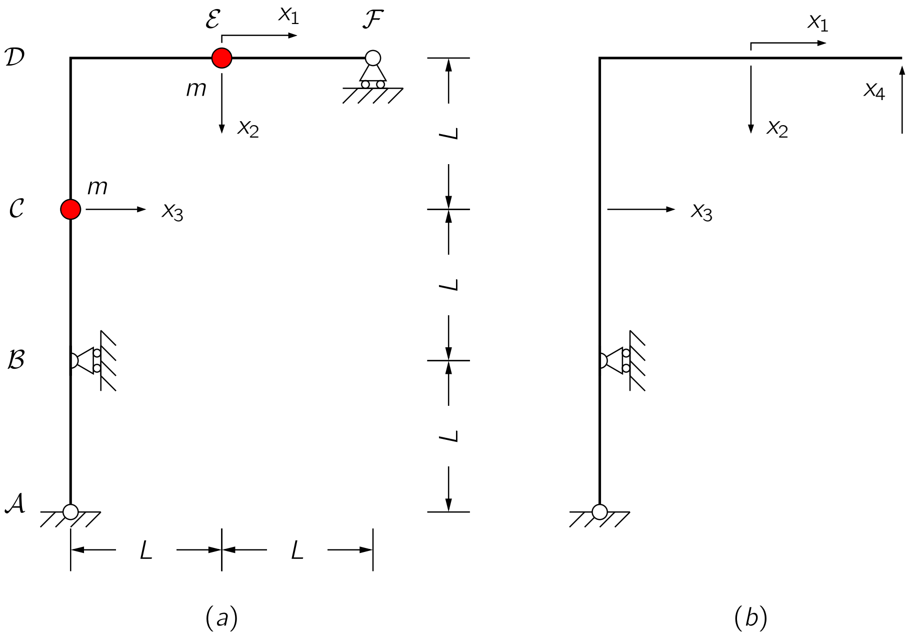
In the first equation above, each θ_n is the *relative* rotation across the spring number n, $n = 1, \ldots, 5$. The relative rotations depend on the nodal displacements, $\theta_n = \theta_n(\boldsymbol{x})$ and, for example, it is

$$\theta_2 = \frac{x_3 - x_2}{L} - \frac{x_2 - x_1}{L} = \frac{x_3 - 2x_2 + x_1}{L}$$

(note that to write the expressions for the other θ's you have to take into account the prescribed boundary conditions).

Finally, substituting $\theta_n(\boldsymbol{x})$ for θ_n in the first expression of V and comparing with the other one you can derive the values of the k_{ij}.

6 3 DOF System



In figure (a) is shown a single, uniform beam (flexural stiffness EJ, unit mass $\bar{m}$) that supports two bodies of equal mass m and it is constrained to the ground in $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{F}$.

Considering that (1) each one of the suspended masses is much greater than the total mass of the beam, $m \gg 5\bar{m}L$ and (2) the beam axial deformations are negligible with respect to the flexural ones, it is possible to use the three displacement components indicated in figure (a) as the dynamical degrees of freedom of the system.

Using the three dynamical degrees of freedom of figure (a)

1: compute the structural matrices $\boldsymbol{M}$ and $\boldsymbol{K}$,

2: compute the eigenvalues and the normalized eigenvectors of the system.

The system, initially at rest, is then subjected to a horizontal ground displacement

$$u_g(t) = \begin{cases} 0 & t < 0 \\ \delta \sin \omega t & t \geq 0 \end{cases}$$

where

$$\delta = \frac{L}{1000} \qquad \text{and} \qquad \omega = \frac{11}{4}\omega_0 = \frac{11}{4}\sqrt{\frac{EJ}{mL^3}},$$

3. write the modal equations of motion and find the modal responses,
4. plot the displacement component $x_3(t)$ in the interval $0 \leq \omega_0 t \leq 10$,
5. plot the support reaction $R_{\mathcal{F}}(t)$ in the interval $0 \leq \omega_0 t \leq 10$.

Hint

One possible way to compute the stiffness matrix consists in (1) releasing the constraint in $\mathcal{F}$, so that we can consider its reaction $R_{\mathcal{F}}$ and display the associated displacement, x_4, as it is shown in figure (b), (2) computing the 4×4 *extended* flexibility matrix $\bar{\boldsymbol{F}}$ associated with the isostatic system of figure (b), (3) writing the displacement components in terms of the nodal and reaction forces, with the understanding that $x_4 \equiv 0$

$$\bar{\boldsymbol{F}} \begin{Bmatrix} \boldsymbol{f}_d \\ R_{\mathcal{F}} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{F}_{dd} & \boldsymbol{F}_{d\mathcal{F}} \\ \boldsymbol{F}_{\mathcal{F}d} & \boldsymbol{F}_{\mathcal{F}\mathcal{F}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{f}_d \\ R_{\mathcal{F}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{x} \\ 0 \end{Bmatrix},$$

(4) deriving $R_{\mathcal{F}}$ from the last equation and substituting its value in the first one

$$R_{\mathcal{F}} = -\boldsymbol{F}_{\mathcal{F}\mathcal{F}}^{-1}\,\boldsymbol{F}_{\mathcal{F}d}\,\boldsymbol{f}_d, \qquad \Rightarrow \qquad \left(\boldsymbol{F}_{dd} - \boldsymbol{F}_{d\mathcal{F}}\,\boldsymbol{F}_{\mathcal{F}\mathcal{F}}^{-1}\,\boldsymbol{F}_{\mathcal{F}d}\right)\boldsymbol{f}_d = \boldsymbol{F}\,\boldsymbol{f}_d = \boldsymbol{x},$$

gives a closed form expression for the flexibility matrix $\boldsymbol{F}$ associated with the dynamical degrees of freedom and the corresponding elastic nodal forces and, finally (5) computing $\boldsymbol{K}$ by inversion of $\boldsymbol{F}$.

The extended flexibility matrix, easily computed using the PVD, is

$$\bar{\boldsymbol{F}} = \frac{L^3}{6EJ}\begin{bmatrix} 24 & 16 & 9 & -32 \\ 16 & 16 & 5 & -33 \\ 9 & 5 & 4 & -10 \\ -32 & -33 & -10 & 72 \end{bmatrix}$$