Earthquake excitation

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Outline

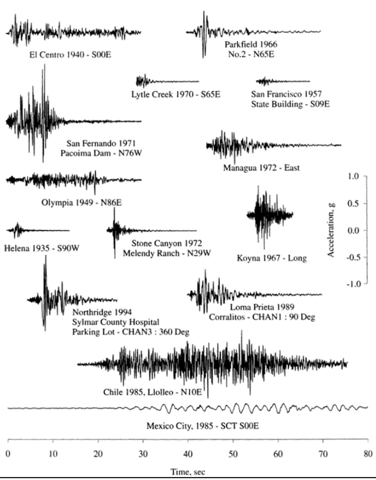
Seismic Excitation

The most important quantity related to earthquake excitation is the ground acceleration.
Ground acceleration can be recorded with an accelerometer, basically a SDOF oscillator, with a damping ratio $\zeta \approx 70\%$, whose displacements are proportional to ground accelerations up to a given frequency.
Instrument records of *strong ground motion* first became available in the '30s, the first record of a destructive ground motion being the 1940 records of El Centro earthquake.

Seismic Excitation

Historically, most of the strong motion records were recorded for a few earthquakes, in California and Japan, in different places and different locations (in the free field, on building foundations, on different building storeys etc), while a lesser number of records were available for different areas.
In more recent years, many national research agencies installed and operated networks of strong motion accelerometers, so that the availability of strong motion records, recorded in different geographic areas and under different local conditions is constantly improving.
Moreover, in many countries the building codes require that important constructions must be equipped with accelerometers, further increasing the number of available records.
`http://peer.berkeley.edu/smcat/search.html`,
`http://peer.berkeley.edu/peer_ground_motion_database`,
`http://itaca.mi.ingv.it/ItacaNet/`.

Seismic Excitation Samples



A number of different strong motion records, recorded at different sites and due to different earthquakes, are plotted with the same scale, both in time and in acceleration.
Appreciate the large variability in terms of amplitudes, duration and frequency content of the different records.
We need a method to categorize this variability.

Detailed Sample

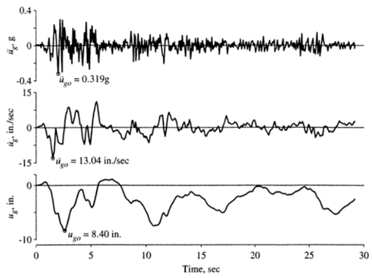


Figure 6.1.4 North–south component of horizontal ground acceleration recorded at the Imperial Valley Irrigation District substation, El Centro, California, during the Imperial Valley earthquake of May 18, 1940. The ground velocity and ground displacement were computed by integrating the ground acceleration.

Above, the acceleration recorded at El Centro during the Imperial Valley 1940 e.q., along with the velocity and displacements obtained by numerical integration.
For meaningful results, the initial conditions of integration and the removal of linear trends from the acceleration record are of capital importance (read: don't try this at home).

Detailed Sample

Strong motion being a very irregular motion, a high number of samples is required to accurately describe it.
Modern digital instruments record the acceleration at a rate of 200 and more samples per second and minimize the need for sophisticated correction of the accelerations before the time integration.

About Ground Motion

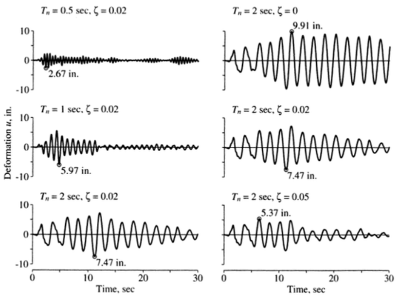
In the following, we consider so called *free-field* records, that is recorded on ground free surface in a position that is deemed free from effects induced by building response.
Tough accelerations vary with time in a very irregular manner, the variation is fully known, and for an individual record we can write the equation of motion in terms of the displacement response function $D(t)$,

$$\ddot{D} + 2\zeta\omega\dot{D} + \omega^2 D = -\ddot{u}_g(t).$$

Clearly, the displacement response function, for assigned $\ddot{u}_g$, depends on ζ and ω only.
Of course, due to the irregular nature of ground excitation the response must be evaluated numerically.
Our first step will be to explore the dependency of D on ω (or rather T_n as it is usual in earthquake engineering) and ζ.

T_n and ζ dependency

Leftmost column, fixed $\zeta = 0.02$ and $T_N = 0.5, 1.0, 2.0$ s. Although the ground motion is irregular, the responses have a similarity, each one having a period close to T_n.



Centre column, fixed $T_n = 2.0$ s and $\zeta = 0, 0.02, 0.05$. For a fixed period the shapes are similar while the maximum response values depends on ζ.

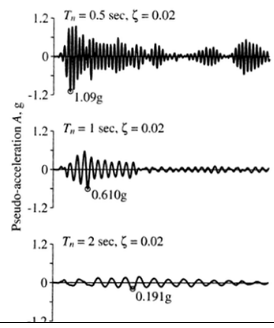
Displacement response functions for the El Centro 1940 NS acceleration record.

Pseudo Acceleration

From deformation response, we can compute the equivalent static force

$$f_s(t) = m\omega^2 D(t) = mA(t)$$

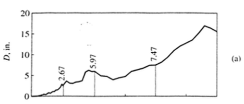
where $A(t)$ is the pseudo acceleration,
$A(t) = ww_n D(t) = (2\pi)^2 D(t)/T_n^2$
Note, ome more time, that f_s is proportional to $A(t)$ and not to the acceleration $\ddot{D}(t)$.



Left, pseudo accelerations computed for varying T_n. Compare with previous page's figure. The relative magnitudes are reverted: for $T_n = 0.5$ s we have a maximum force and a minimum displacement, while for $T_n = 2.0$ s the force is minimum and the displacement maximum.

Response Spectrum

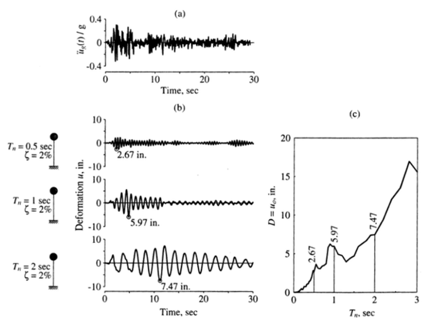
Introduced by M.A. Biot in 1932, popularised by G.W. Housner, the concept of response spectrum is fundamental to characterise e.q. response.



The response spectrum is a plot of the peak values of a response quantity, say the displacement response function, computed for different values of T_n and the same ζ, versus natural period T_N.

A graph where several such plots, obtained for different values of ζ, representative of different damping ratios that characterise different structures, are plotted close to each other represents the e.q. characteristics from the point of view of peak structural response (wait later slides for examples).

Computing the DRS



For a fixed values of ζ (usually one of 0% 0.5% 1% 2% 3% 5% 7% 10% 15% and 20%) and for variable values of T_n (usually ranging from 0.01 s to 20 s)

1. the displacement response function is numerically integrated,
2. the peak value is individuated,
3. the peak value is plotted.

Pseudo Spectra

Only the Deformation Response Spectrum (DRS) is required to fully characterise the peaks of deformations and equivalent static forces.
It is however useful to study also the pseudo acceleration (PARS) and pseudo velocity (PVRS) spectra, as they are useful in understanding excitation intrinsic characteristics, in constructing *design spectra* and to connect dynamics and building codes.
We have already introduced $A(t)$, consider now the quantity

$$V(t) = \omega_n D(t) = \frac{2\pi}{T_n} D(t)$$

that is, the pseudo velocity.
The peak value of V is connected with the maximum strain energy,

$$E_{s,0} = \frac{1}{2} m V_0^2$$

being $E_{s,0} = \frac{1}{2} D_0\, m\omega^2 D_0$. Once again, $V \neq \dot{x}$, the relative velocity.

Pseudo Spectra

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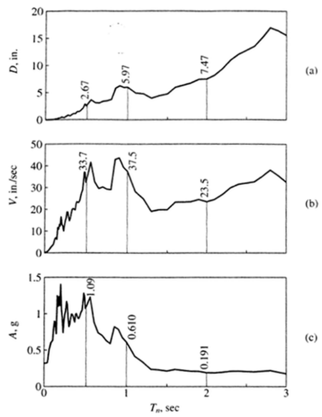
Earthquakes and Earthquake Response
Response Spectrum
Combined $D-V-A$ spectrum
Tripartite Plots
Example of Use of the Spectrum
Response Spectrum Characteristics



Deformation spectrum, pseudo velocity and pseudo acceleration spectra for El Centro 1940 NS, $\zeta = 2\%$.

Combined $D - V - A$ spectrum

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Earthquakes and Earthquake Response
Response Spectrum
Combined $D-V-A$ spectrum
Tripartite Plots
Example of Use of the Spectrum
Response Spectrum Characteristics

In the following, we will use the symbols D, V and A to represent the values of the DRS, PVRS and PARS spectra, respectively, with

$$V = \omega_n D, \qquad A = \omega_n^2 D$$

While D, V and A represent the same information, nonetheless it is useful to maintain a distinction as they are connected to different response quantities, the maximum deformation, the maximum strain energy and the maximum equivalent static force.
Moreover, it is possible to plot all three spectra on the same logarithmic plot, giving what is regarded as a fundamental insight into the ground motion characteristics.

Constant A

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Combined $D-V-A$ spectrum
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Response Spectrum Characteristics

Consider a plane with axes $\log T_n$ and $\log V$, and the locus of this plane where A is constant, $A = \hat{A}$:
it is

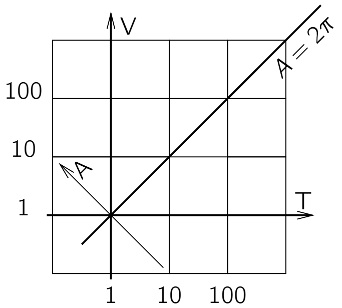
$$A = 2\pi V / T_n = \hat{A}$$

taking the logarithm

$$\log \frac{\hat{A}}{2\pi} = \log V - \log T_n$$

or

$$\log V = \log T_n + \log \frac{\hat{A}}{2\pi}.$$



In the log-log plane straight lines at 45° are characterised by a constant value of A.

Constant D

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Earthquakes and Earthquake Response
Response Spectrum
Combined $D-V-A$ spectrum
Tripartite Plots
Example of Use of the Spectrum
Response Spectrum Characteristics

In the same plane with axes $\log T_n$ and $\log V$ we seek the locus where D is constant, $D = \hat{D}$:
it is

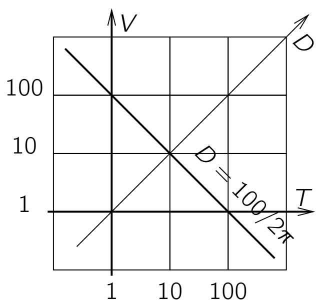
$$D = T_n V / 2\pi = \hat{D}$$

taking the logarithm

$$\log 2\pi\, \hat{D} = \log V + \log T_n$$

or

$$\log V = \log 2\pi\, \hat{D} - \log T_n.$$



In the log-log plane straight lines at −45° are characterised by a constant value of D.

Example of Construction, 1

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Earthquakes and Earthquake Response
Response Spectrum
Combined $D-V-A$ spectrum
Tripartite Plots
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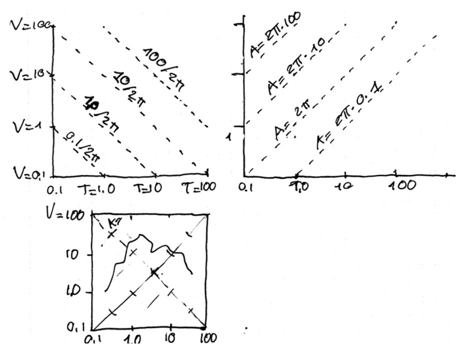
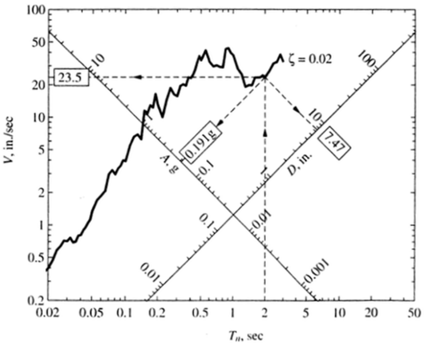
Response Spectrum Characteristics

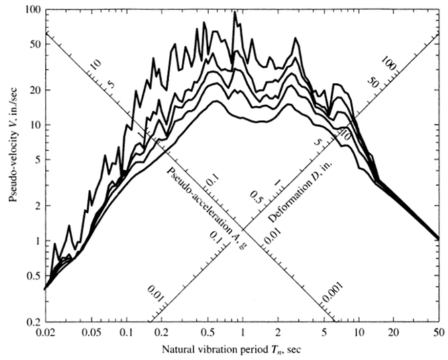


Example of Construction, 2



Combined $D-V-A$ response spectrum, El Centro 1940 NS record, $\zeta = 0.02$.

Example of $D-V-A$ spectrum



Combined $D-V-A$ response spectrum, El Centro 1940 NS record,

for $0 \le \zeta \le 20\%$ and full range of periods.

Peak Structural Response

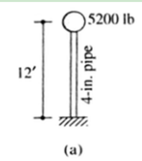
The peak deformation u_0 is given by

$$u_0 = D$$

and the peak of the equivalent static force $f_{S,0}$ is given by

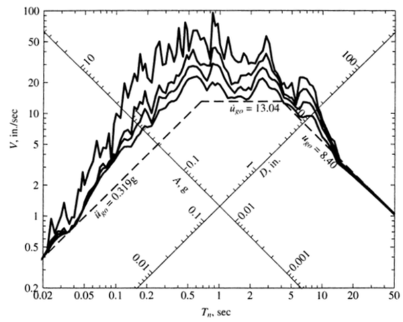
$$f_{S,0} = k u_0 = m\omega^2 u_0 = kD = mA$$

It is required to know the peak of the base bending moment for the structure on the right, when subjected to the NS component of the El Centro 1940 record.



The mass is $m = 2360\,\text{kg}$, the stiffness is $k = 36.84\,\text{kN}\,\text{m}^{-1}$, the natural period of vibration is computed as $T_n = 1.59\,\text{s}$. The damping ratio is assumed to be $\zeta = 5\%$.
On the graph of the relevant $D-V-A$ spectrum, for $T_n = 1.59$, we find the value $A = 0.20\,\text{g}$.
The equivalent static force is $f_{S,0} = 2360\,\text{kg} \cdot 0.20 \cdot 9.81\,\text{m}\,\text{s}^{-2} = 4.63\,\text{kN}$ and the peak base bending moment is
$M_{b,0} = 4.63\,\text{kN} \cdot 12 \cdot 0.305\,\text{m} = 16.93\,\text{kN}\,\text{m}$

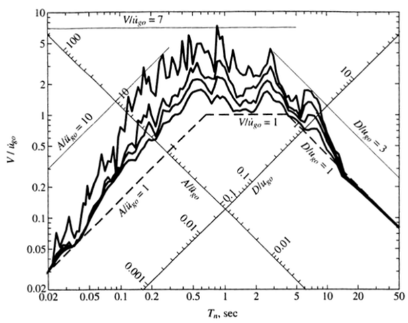
Response Spectrum Characteristics



For intermediate values of T_n it is apparent that

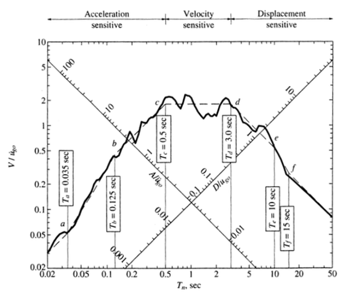
- $A > \ddot{u}_{g,0}$, $V > \dot{u}_{g,0}$ and $D > u_{g,0}$;
- $V_{\max} \approx$ constant for each value of ζ,
- there is a clear dependency on ζ.

Idealised Response Spectrum, 1



First step in the construction of an idealised $D-V-A$ response spectrum is to make a tripartite plot with all three ordinate axes normalised with respect to $u_{g,0}$, $\dot{u}_{g,0}$ and $\ddot{u}_{g,0}$.

Idealised Response Spectrum, 1



Next,

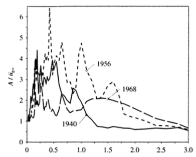
- draw the $\zeta = 5\%$ spectrum,
- individuate the intervals where a) $A \approx \ddot{u}_{g,0}$, b) $A \approx a_A \ddot{u}_{g,0}$, c) $V \approx a_V \dot{u}_{g,0}$, d) $D \approx a_D u_{g,0}$, e) $D \approx u_{g,0}$ and
- individuate approximate amplifications factors, a_A, a_V and a_D,
- connect the constant value intervals with straight lines.

Idealised Response Spectrum

Our procedure results look good in the log-log graph, but should we represent the same piecewise linearisation in a lin-lin graph it will be apparent that's a rather crude approximation.
This consideration is however not particularly important, because we are not going to use the idealised spectrum in itself, but as a guide to help developing design spectra.
Finally, consider that the positions of the points $T_a, \ldots, T_f$ and the amplifications factors a_A, a_V and a_D are not equal for spectra of different earthquakes recorded at different sites, they depend in complex and not fully determined ways on different parameters, for example the focal distance and the focal mechanism and, very important, the local soil characteristics, showing in the whole a large variability.

Elastic Design Spectra

On the right, the $\ddot{u}_{g,0}$ normalised A response spectra for 3 different earthquakes NS records, recorded at the same El Centro site.
Clearly, it is not possible to infer the jagged appearance of the 1968 spectra from the 1940's and 1956's ones.



For design purposes, however, it is not necessary to know in advance and in detail the next quake's response spectra as it suffices to know some sort of an upper bound on spectral ordinates, that is a *Design Spectrum*.

Elastic Design Spectra

A design spectrum is usually specified as an idealized response spectrum, as a set of connected straight lines on the log-log $D-V-A$ plot, and has not, in contrast with a response spectrum, a jagged appearance.
Note that straight lines on a log-log graph map on straight or curved lines on conventional $T-n$—A plots.

The requirements of a design spectrum are manifold, but mostly important a design spectrum must be an envelope of possible peak values.

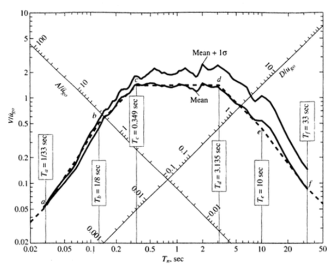
Elastic Design Spectra

The procedure used for computing an elastic design spectrum could be sketched as follows,

- collect earthquake records from the site under study or from similar sites (similar in local geology, in epicentral distances, duration of strong motion etc) and compute normalized response spectra,
- statistically characterise, in terms of mean values and standard deviations, the set of normalised spectral ordinates at hand,
- derive idealized spectra.

Derivation of an Elastic Design Spectra

Riddel and Newmark (1979)



Riddel and Newmark

a collected a large set of records for similar sites in Southern California,

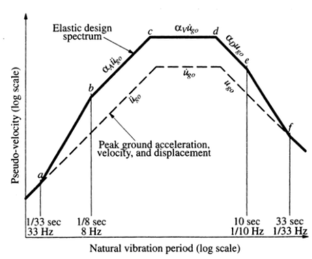
b computed the normalised response spectra for $z = 5\%$ and, finally

c computed the mean value and the standard deviation of the peak response distribution.

In the graph, the summary of their research: the mean and mean+1σ spectra for 5% damping ratio.
In the same graph, you can see also (dashed) an idealised spectrum representation of the mean spectrum.

Idealised Elastic Design Spectra

It is common practice to subdivide the design $D-V-A$ elastic spectrum in 7 segments and use 4 key vibration periods, together with given amplification factors, to draw the required idealised design spectrum.



The key periods $T_a = 0.03\,\mathrm{s}$ and $T_b = 0.125\,\mathrm{s}$ define the segment where A rises from 1 to α_A
The key periods $T_e = 10\,\mathrm{s}$ and $T_f = 33\,\mathrm{s}$ define the segment where D decreases from α_D to 1.
The key periods T_c and T_d, instead, follows from applying the given amplification factors to pseudo accelerations, pseudo velocities and deformation.

Example Data

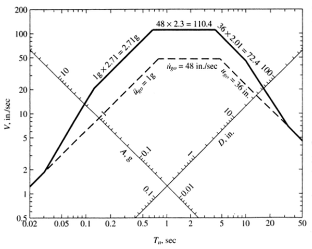
	Median (50th percentile)			Median+1σ (84th percentile)		
ζ (%)	α_A	α_V	α_D	α_A	α_V	α_D
1	3.21	2.31	1.82	4.38	3.38	2.73
2	2.74	2.03	1.63	3.66	2.92	2.42
5	2.12	1.65	1.39	2.71	2.30	2.01
10	1.64	1.37	1.20	1.99	1.84	1.69
20	1.17	1.08	1.01	1.26	1.37	1.38

	Median	Median+1σ
α_A	$3.21 - 0.68\log\zeta$	$4.38 - 1.04\log\zeta$
α_V	$2.31 - 0.41\log\zeta$	$3.38 - 0.67\log\zeta$
α_D	$1.82 - 0.27\log\zeta$	$2.73 - 0.45\log\zeta$

Source: N.M. Newmark and W.J. Hall, *Earthquake Spectra and Design*, EERC Report 1982.

Procedure Summary

1. for the site in case, get an estimate of $\ddot u_{g,0}$, $\dot u_{g,0}$ and $u_{g,0}$, from an analysis of relevant data or desuming it from literature,
2. in the tripartite log-log graph, draw a line for each of the shaking parameters,
3. for a selected value of ζ amplify the shaking parameters by appropriate amplification factor and draw a line for each amplified parameter,
4. draw vertical lines from the key periods to individuate the connection ramps,
5. draw the idealised design spectrum.

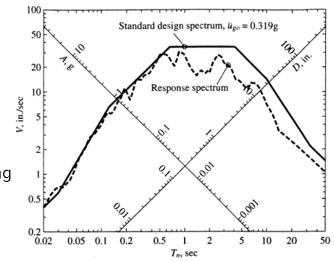


A common assumption, used when only the $\ddot u_{g,0}$ estimate is available, is

$$\dot u_{g,0} = \ddot u_{g,0}\frac{120\,\mathrm{cm\,s^{-1}}}{g} \quad\text{and}\quad u_{g,0} = \ddot u_{g,0}\frac{90\,\mathrm{cm}}{g}.$$

Comparison of Design and Response Spectra, 1

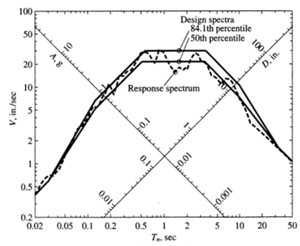
In the figure, the response spectrum for the 1940 El Centro NS acceleration record, computed for $\zeta = 5\%$, and the corresponding design spectrum, with amplifications corresponding to median values of the ordinates.



The spectrum was constructed from the real value of $\ddot u_{g,0} = 0.319\,g$ and estimated values of $\dot u_{g,0} = \ddot u_{g,0}\frac{48\,\mathrm{inch/s}}{g} = 15.3\,\mathrm{inch/s}$ and $u_{g,0} = \ddot u_{g,0}\frac{90\,\mathrm{cm}}{g} = 11.5\,\mathrm{inch}$, estimated values that are significantly higher than the effective values.
There is a good concordance in the acceleration controlled part of the design spectrum, but spectral velocities and deformations are not very good, due to rather poor estimates of the relevant ground motion peak quantities.

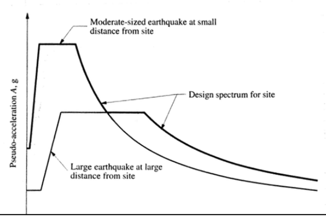
Comparison of Design and Response Spectra, 2

In this second slide the design spectra are two, the median and the median $+ 1\sigma$ versions, both based on *exact* peak values of the ground motion. While the median spectrum is, ok, in a median position with respect to the ordinates of the elastic response spectrum, the presumed envelope spectrum does effectively a good job, maxing out most of the spikes present in the elastic response spectrum.



Differences between Response and Design Spectra

The response spectrum is a description, in terms of its peak effects, of a particular ground motion.
The design spectrum is a specification, valid for a site or a class of sites, of design seismic forces.
If a site falls in two different classifications, e.g., the site is near to a seismic fault associated with low magnitude earthquakes and it is distant from a fault associated with high magnitude earthquakes, with the understanding that the frequency contents of the two classes of events are quite dissimilar the design spectrum should be derived from the superposition of the two design spectra.



Earthquake Response of Inelastic Systems

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June 11, 2013

Outline

Motivation

In Earthquake Engineering it is common practice to design against a large earthquake, that has a given mean period of return (say 500 years), quite larger than the expected life of the construction.

A period of return of 500 years means that in a much larger interval, say 50000 years, you expect say 100 earthquakes that are no smaller (in the sense of some metrics, e.g., the peak ground acceleration) than the design earthquake.

Motivation

If you know the peak ground acceleration associated with the design earthquake, you can derive elastic design spectra and then, from the ordinates of the pseudo-acceleration spectrum, derive equivalent static forces to be used in the member design procedure.
However, in the almost totality of cases the structural engineer does not design the anti-seismic structures considering the ordinates of the elastic spectrum of the maximum earthquake, the preferred procedure is to reduce these ordinates by factors that can be as high as 6 or 8. This, of course, leads to a large reduction in the cost of the structure.

Motivation

If we design for forces smaller than the forces likely to occur during a large earthquake, our structures will be damaged, or even destroyed.
The reasoning behind such design procedure is that, for the unlikely occurrence of a large earthquake, a large damage in the construction is acceptable as far as no human lives are taken in a complete structural collapse and that, in the mean, the costs for repairing a damaged building are not disproportionate to its value.

What to do?

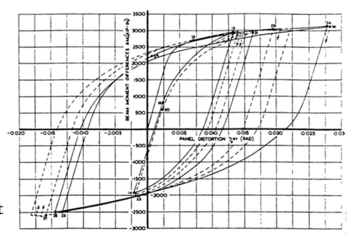
To ascertain the amount of acceptable reduction of earthquake loads it is necessary to study

- the behaviour of structural members and systems subjected to cyclic loading outside the elastic range, to understand the amount of plastic deformation and cumulated plastic deformation that can be sustained before collapse and
- the global structural behaviour for inelastic response, so that we can relate the reduction in design ordinates to the increase in members' plastic deformation.

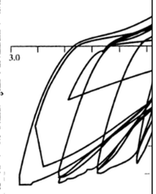
The first part of this agenda pertains to Earthquake Engineering proper, the second part is across EE and Dynamics of Structures, and today's subject.

Cyclic behaviour

Investigation of the cyclic behaviour of structural members, sub-assemblages and scaled or real sized building model, either in labs or via numerical simulations, constitutes the bulk of EE. What is important, at the moment, is the understanding of how different these behaviours can be, due to different materials or structural configurations, with instability playing an important role.

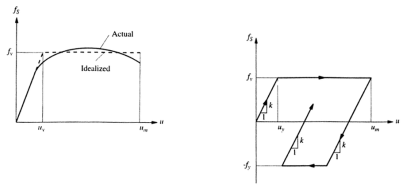


(a)



We will see 3 different diagrams, force vs deformation, for a clamped steel beam subjected to flexure, a reinforced concrete sub-assemblage an a masonry wall.

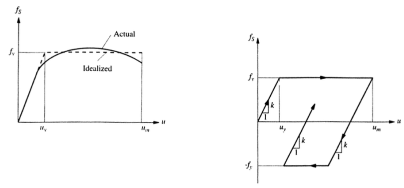
E-P model



A more complex behaviour may be represented with an elasto-perfectly plastic (e-p) bilinear idealisation, see figure, where two important requirements are obeyed

1. the initial stiffness of the idealised e-p system is the same of the real system, so that the natural frequencies of vibration for small deformation are equal,
2. the yielding strength is chosen so that the sum of stored and dissipated energy in the e-p system is the same as the energy stored and dissipated in the real system.

E-P model, 2



In perfect plasticity, when yielding (a) the force is constant, $f_S = f_y$ and (b) the stiffness is null, $k_y = 0$. The force f_y is the yielding force, the displacement $x_y = f_y/k$ is the yield deformation.
In the right part of the figure, you can see that at unloading ($dx = 0$) the stiffness is equal to the initial stiffness, and we have $f_s = k(x - x_{\text{ptot}})$ where x_{ptot} is the total plastic deformation.

Definitions

For a given seismic excitation, we give the following definitions

equivalent system a linear system with the same characteristics (ω_n, ζ) of the non-linear system

normalised yield strength, $\bar f_y$ is the ratio of the yield strength to the peak force of the equivalent system,

$$\bar f_y = \min\left\{\frac{f_y}{f_0} = \frac{x_y}{x_0},\, 1\right\}.$$

It is $\bar f_y \le 1$ because for $f_y \ge f_0$ there is no yielding, and in such case we define $\bar f_y = 1$.

yield strength reduction factor, R_y it comes handy to define R_y, as the reciprocal of $\bar f_y$,

$$R_y = \frac{1}{\bar f_y} = \max\left\{1,\, \frac{f_0}{f_y} = \frac{x_0}{x_y}\right\}.$$

normalised spring force, $\bar f_S$ the ratio of the e-p spring force to the yield strength,

$$\bar f_S = \frac{f_S}{f_y}.$$

Definitions, cont.

equivalent acceleration, a_y the (pseudo-)acceleration required to yield the system,
$a_y = \omega_n^2 x_y = f_y/m$.

e-p peak response, x_m the elasto-plastic peak response

$$x_m = \max_t \{|x(t)|\}.$$

ductility factor, μ (or ductility ratio) the normalised value of the e-p peak response

$$\mu = \frac{x_m}{x_y}.$$

Whenever it is $R_y > 1$ it is also $\mu > 1$.

Note that the ratio between the e-p and elastic peak responses is given by

$$\frac{x_m}{x_0} = \frac{x_m}{x_y}\frac{x_y}{x_0} = \mu\,\bar f_y = \frac{\mu}{R_y} \;\rightarrow\; \mu = R_y\frac{x_m}{x_0}.$$

Normalising the force

For an e-p system, the equation of motion (*EOM*) is

$$m\ddot x + c\dot x + f_S(x, \dot x) = -m\ddot u_g(t)$$

with f_S as shown in a previous slide. The *EOM* must be integrated numerically to determine the time history of the e-p response, $x(t)$.
For a given excitation $\ddot x_g(t)$, the response depends on 3 parameters, $\omega_n = \sqrt{k/m}$, $\zeta = c/(2\omega_n m)$ and x_y.
If we divide the *EOM* by m, recalling our definition of the normalised spring force, the last term is

$$\frac{f_S}{m} = \frac{1}{m}\frac{f_y}{f_y}f_S = \frac{1}{m}k\,x_y\frac{f_S}{f_y} = \omega_n^2 x_y \bar f_S$$

and we can write

$$\ddot x + 2\zeta\omega_n\dot x + \omega_n^2 x_y \bar f_S(x, \dot x) = -\ddot u_g(t)$$

Normalising the displacements

With the position $x(t) = \mu(t)\,x_y$, substituting in the *EOM* and dividing all terms by x_y, it is

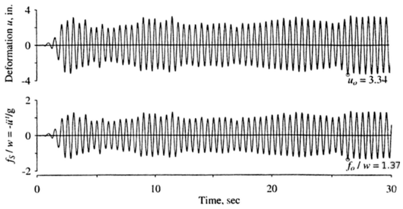
$$\ddot\mu + 2\omega_n\zeta\dot\mu + \omega_n^2\bar f_S(\mu, \dot\mu) = -\frac{\omega_n^2}{\omega_n^2}\frac{\ddot x_g}{x_y} = -\omega_n^2\frac{\ddot x_g}{a_y}$$

It is now apparent that the input function for the ductility response is the acceleration ratio: doubling the ground acceleration or halving the yield strength leads to exactly the same response $\mu(t)$ and the same peak value μ.
The equivalent acceleration can be expressed in terms of the normalised yield strength $\bar f_y$,

$$a_y = \frac{f_y}{m} = \frac{\bar f_y\, f_0}{m} = \frac{\bar f_y\, k x_0}{m} = \bar f_y\,\omega_n^2\,x_0$$

and recognising that x_0 depends only on ζ and ω_n we conclude that, for given $\ddot x_g(t)$ and $\bar f_S(\mu, \dot\mu)$ the ductility response depends only on ζ, ω_n, $\bar f_y$.

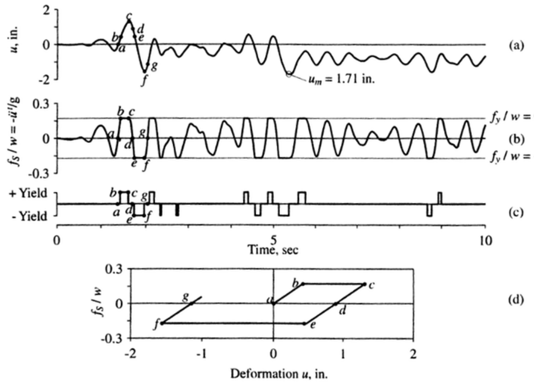
Elastic response, required parameters



Response of linear system with $T_n = 0.5$ sec and $\zeta = 0$ to El Centro

In the figure above, the elastic response of an undamped, $T_n = 0.5\,$s system to the NS component of the El Centro 1940 ground motion (all our examples will be based on this input motion).
Top, the deformations, bottom the elastic force normalised with respect to weight, from the latter peak value we know that all e-p systems with $f_y < 1.37$w will experience plastic deformations during the EC1940NS ground motion.

Inelastic response, $\bar{f}_y = 1/8$



The various response graphs above were computed for $\bar{f}_y = 0.125$ (i.e., $R_y = 8$ and $f_y = \frac{1.37}{8}\text{w} = 0.171\text{w}$) and $\zeta = 0$, $T_n = 0.5\,$s.

Inelastic response, $\bar{f}_y = 1/8$

The force-deformation diagram for the first two excursions in plastic domain, the time points a, b, c, d, e, f and g are the same in all 4 graphs:

- until $t = b$ we have an elastic behaviour,
- until $t = c$ the velocity is positive and the system accumulates positive plastic deformations,
- until $t = e$ we have an elastic unloading (note that for $t = d$ the force is zero, the deformation is equal to the total plastic deformation),
- until $t = f$ we have another plastic excursion, cumulating negative plastic deformations
- until at $t = f$ we have an inversion of the velocity and an elastic reloading.

Response for different $\bar{f}_y$'s

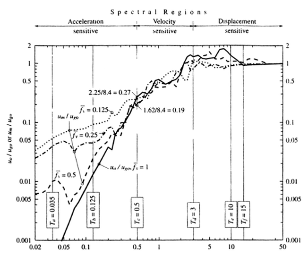
$\bar{f}_y$	x_m	x_{perm}	μ
1.000	2.25	0.00	1.00
0.500	1.62	0.17	1.44
0.250	1.75	1.10	3.11
0.125	2.07	1.13	7.36

This table was computed for $T_n = 0.5\,$s and $\zeta = 5\%$ for the EC1940NS excitation.
Elastic response was computed first, with peak response $x_0 = 2.25\,$in and peak force $f_0 = 0.919$w, later the computation was repeated for $\bar{f}_y = 0.5,\ 0.25,\ 0.125$.
In our example, all the peak values of the e-p responses are smaller than the elastic one, but this is not always true, and shouldn't be generalised.
The permanent displacements increase for decreasing yield strengths, and also this fact shouldn't be generalised.
Last, the ductility ratios increase for decreasing yield strengths, for our example it is $\mu \approx R_y$.

Ductility demand and capacity

We can say that, for a given value of the normalised yield strength $\bar{f}_y$ or of the yield strength reduction factor R_y, there is a *ductility demand*, a measure of the extension of the plastic behaviour that is required when we reduce the strength of the construction.
Corresponding to this ductility demand our structure must be designed so that there is a sufficient *ductility capacity*.
Ductility capacity is, in the first instance, the ability of individual members to sustain the plastic deformation demand without collapsing, the designer must verify that the capacity is greater than the demand for all structural members that go non linear during the seismic excitation.

Effects of T_n

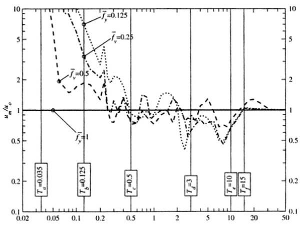


$x_m/x_{g,0}$

For EC1940NS, for $\zeta = .05$, for different values of T_n and for $\bar{f}_y = 1.0,\ 0.5,\ 0.25,\ 0.125$ the peak response x_0 of the equivalent system (in black) and the peak responses of the 3 inelastic systems has been computed.

There are two distintinct zones: left there is a strong dependency on $\bar{f}_y$, the peak responses grow with R_y; right the 4 curves intersects with each other and there is no clear dependency on $\bar{f}_y$.

Effects of T_n

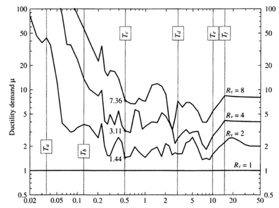


x_m/x_0

With the same setup as before, here it is the ratio of the x_m's to x_0, what is evident is the fact that, for large T_n, this ratio is equal to 1... this is justified because, for large T_n's, the mass is essentially at rest, and the deformation, either elastic or elasto-plastic, are equal and opposite to the ground displacement.

Also in the central part, where elastic spectrum ordinates are dominated by the ground velocity, there is a definite tendency for the x_m/x_0 ratio, that is $x_m/x_0 \approx 1$

Effects of T_n



μ

With the same setup as before, in this graph are reported the values of the ductility factor μ. The values of μ are almost equal to R_y for large values of T_n, and in the limit, for $T_n \to \infty$, there is a strict equality. An even more interesting observation regard the interval $T_c \le T_n \le T_f$, where the values of μ oscillate near the value of R_y.

On the other hand, the behaviour is completely different in the acceleration controlled zone, where μ grows very fast, and the ductility demand is very high even for low values (0.5) of the yield strength reduction factor.

The results we have discussed are relative to one particular excitation, nevertheless research and experience confirmed that these propositions are true also for different earthquake records, taking into account the differences in the definition of spectral regions.

Response Spectrum for Yield States

The first step in an anti seismic design is to set an available ductility (based on materials, conception, details).
In consequence, we desire to know the yield displacement u_y or the yield force f_y

$$f_y = k u_y = m\omega_n^2 u_y$$

for which the ductility demand demand imposed by the ground motion is not greater than the available ductility.

Response Spectrum for Yield States

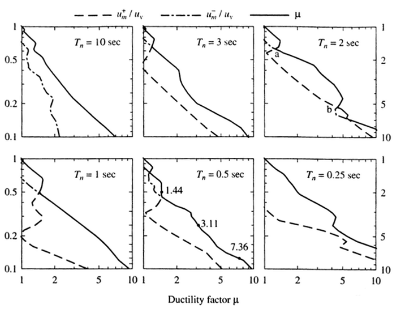
For each T_n, ζ and μ, the *Yeld-Deformation Response Spectrum* (D_y) ordinate is the corresponding value of u_y: $D_y = u_y$. Following the ideas used for Response and Design Spectra, we define $V_y = \omega_n u_y$ and $A_y = \omega_n^2 u_y$, that we will simply call pseudo-velocity and pseudo-acceleration spectra. Using our definitions of pseudo acceleration, we can find a more significant expression for the design force:

$$f_y = k u_y = m\omega_n^2 u_y = m A_y = w\frac{A_y}{g},$$

where w is the weight of the structure.

Our definition of inelastic spectra is compatible with the definition of elastic spectra, because for $\mu = 1$ it is $u_y = u_0$. Finally, the D_y spectrum and its derived pseudo spectra can be plotted on the tripartite log-log graph.

Computing D_y

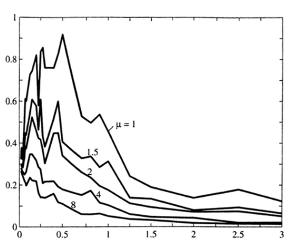


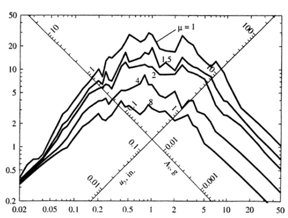
On the left, for different T_n's and $\mu = 5\%$. the independent variable is in the ordinates, either $\bar{f}_y$ (left) or R_y (rigth) the strength reduction factor. Dash-dash lines is u_m^+/u_y, dash-dot is u_m^-/u_y.

u_m^+ and u_m^- are the peaks of positive and negative displacements of the inelastic system, the maximum of their ratios to u_y is the ductility μ.
If we look at these graphs using μ as the independent variable, it is possible that for a single value of μ there are different values on the tick line: in this case, for security reasons, the designer must design for the higher value of $\bar{f}_y$.

Example

For EC1940NS, $z = 5\%$, the yield-strength response spectra for $\mu = 1.0,\ 1.5,\ 2.0,\ 4.0,\ 8.0$.



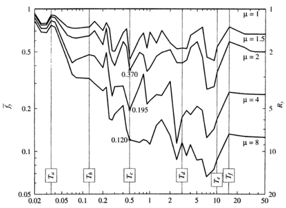


On the left, a lin-lin plot of the pseudo-acceleration normalized (and adimensionalised) with respect to g, the acceleration of gravity.

On the right, a log-log tripartite plot of the same spectrum. Even a small value of μ produces a significant reduction in the required strength.

$\bar{f}_y$ vs μ

We have seen that $\bar{f}_y = \bar{f}_y(\mu, T_n, \zeta)$ is a monotonically increasing function of μ for fixed T_n and ζ.

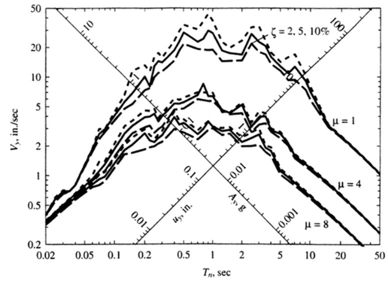


Left the same spectra of the previous slides, plotted in a different format, $\bar{f}_y$ vs T_n for different values of μ.

The implication of this figure is that an anti seismic design can be based on strength, ductility or a combination of the two.

For $T_n = 1.0$, the peak force for EC1940NS in an elastic sistem is $f_0 = 0.919\,\mathrm{w}$, so it is possible to design for $\mu = 1.0$, hence $f_y = 0.919\,\mathrm{w}$ or for an high value of ductility, $\mu = 8.0$, hence $f_y = 0.120 \cdot 0.919\,\mathrm{w}$ or, if such an high value of ductility cannot be easily reached, design for $\mu = 4.0$ and a yielding force of 0.195 times f_0.

Yelding and Damping



El Centro 1940 NS, elastic response spectra and inelastic spectra for $\mu = 4$ and $\mu = 8$, for different values of ζ (2%, 5% and 10%).
Effects of damping are relatively important and only in the velocity controlled area of the spectra, while effects of ductility are always important except in the high frequency range.
Overall, the ordinates reduction due to modest increases in ductility are much stronger than those due to increases in damping.

Energy Dissipation

$$\int^{x(t)} m\ddot{x}\,\mathrm{d}x + \int^{x(t)} c\dot{x}\,\mathrm{d}x + \int^{x(t)} f_S(x,\dot{x})\,\mathrm{d}x = -\int^{x(t)} m\ddot{x}_g\,\mathrm{d}x$$

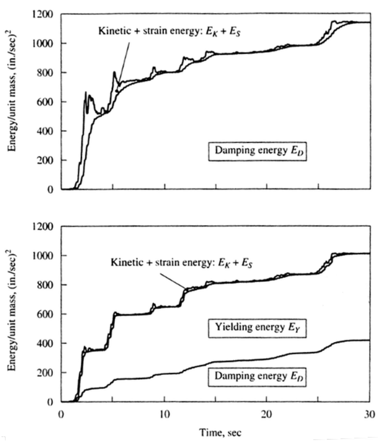
This is an energy balance, between the input energy $\int m\ddot{x}_g$ and the sum of the kinetic, damped, elastic and dissipated by yielding energy.
In every moment, the elastic energy $E_S(t) = \frac{f_S^2(t)}{2k}$ so the yielded energy is

$$E_y = \int f_S(x,\dot{x})\,\mathrm{d}x - \frac{f_S^2(t)}{2k}.$$

The damped energy can be written as a function of t, as $\mathrm{d}x = \dot{x}\,\mathrm{d}t$:

$$E_D = \int c\,\dot{x}^2(t)\,\mathrm{d}t$$

Energy Dissipation



For a system with $m = 1$ and
a) $\bar{f}_y = 1$
b) $\bar{f}_y = 0.25$
the energy contributions during the EC1940NS, $T_n = 0.5\,\mathrm{s}$ and $\zeta = 5\%$.
In a), input energy is stored in kinetic+elastic energy during strong motion phases and is subsequently dissipated by damping.
In b), yielding energy is dissipated by means of some structural damage.

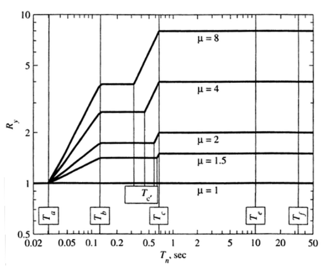
Inelastic Design Spectra

Two possible approaches:

1. compute response spectra for constant ductility demand for many consistent records, compute response parameters statistics and derive inelastic design spectra from these statistics, as in the elastic design spectra procedures;
2. directly modify the elastic design spectra to account for the ductility demand values.

The first procedure is similar to what we have previously seen, so we will concentrate on the second procedure, that it is much more used in practice.

R_y — μ — T_n equations

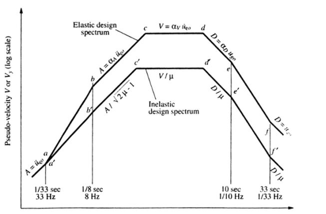


Based on observations and energetic considerations, the plots of R_y vs T_n for different μ values can be approximated with straight lines in a log-log diagram, where the constant pieces are defined in terms of the key periods in $D - V - A$ graphs.

$$R_y = \begin{cases} 1 & T_n < T_a \\ \sqrt{2\mu - 1} & T_b < T_n < T_{c'} \\ \mu & T_c < T_n \end{cases}$$

The key period $T_{c'}$ is different from T_c, as we will see in the next slide; the costant pieces are joined with straight lines in the log-log diagram.

Construction of Design Spectrum



Start from a given elastic design spectrum, defined by the points a-b-c-d-e-f.
Choose a value μ for the ductility demand.
Reduce all ordinates right of T_c by the factor μ, reduce the ordinates in the interval $T_b < T_n < T_c$ by $\sqrt{2\mu-1}$.
Draw the two lines $A = \frac{\alpha_A \ddot{x}_{g0}}{\sqrt{2\mu-1}}$ and $A = \frac{\alpha_V \dot{x}_{g0}}{\mu}$, their intersection define the key point $T_{c'}$.
Connect the point $(T_a, A = \ddot{x}_{g0})$ and the point $(T_b, A = \frac{\alpha_V \dot{x}_{g0}}{\mu})$ with a straight line.
As we already know (at least in principles) the procedure to compute the elastic design spectra for a given site from the peak values of the ground motion, using this simple procedure it is possible to derive the inelastic design spectra for any ductility demand level.

Important Relationships

Earthquake Response of Inelastic Systems
Giacomo Boffi
Motivation
Cyclic Behaviour
E-P Idealisation
Earthquake Response of E-P Systems
Effects of Yielding
Inelastic Response, different values of f_y

For different zones on the T_n axis, the simple relationships we have previuosly defined can be made explicit using the equations that define R_y, in particular we want relate u_m to u_0 and f_y to f_0 for the elastoplastic system and the equivalent system.

1. region $T_n < T_a$, here it is $R_y = 1.0$ and consequently
$$u_m = \mu u_0 \qquad f_y = f_0.$$
2. region $T_b < T_n < T_{c'}$, here it is $R_y = \sqrt{2\mu - 1}$ and
$$u_m = \frac{\mu}{\sqrt{2\mu-1}} u_0 \qquad f_y = \frac{f_0}{\sqrt{2\mu-1}}$$
3. region $T_c < T_n$, here it is $R_y = \mu$ and
$$u_m = u_0 \qquad f_y = \frac{f_0}{\mu}.$$

Similar equations can be established also for the inclined connection segments in the R_y vs T_n diagram.

Application: design of a SDOF system

Earthquake Response of Inelastic Systems
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Motivation
Cyclic Behaviour
E-P Idealisation
Earthquake Response of E-P Systems
Effects of Yielding
Inelastic Response, different values of f_y

- Decide the available ductility level μ (type of structure, materials, details etc).
- Preliminar design, m, k, ζ, ω_n, T_n.
- From an inelastic design spectrum, for known values of ζ, T_n and μ read A_y.
- The design yield strenght is
$$f_y = mA_y.$$
- The design peak deformation, $u_m = \mu D_y/R_y$, is
$$u_m = \frac{\mu}{R_y(\mu, T_n)} \frac{A_y}{\omega_n^2}.$$

Example

Earthquake Response of Inelastic Systems
Giacomo Boffi
Motivation
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E-P Idealisation
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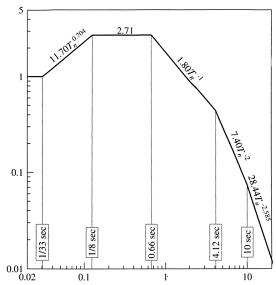
Effects of Yielding
Inelastic Response, different values of f_y



One storey frame, weight w, period is $T_n = 0.25\,$s, damping ratio is $\zeta = 5\%$, peak ground acceleration is $\ddot{x}_{g0} = 0.5\,$g.
Find design forces for
1) system remains elastic,
2) $\mu = 4$ and 3) $\mu = 8$.
In the figure, a reference elastic spectrum for $\ddot{x}_{g0} = 1\,$g, $A_y(0.25) = 2.71\,$g; for $\ddot{x}_{g0} = 0.5\,$g it is $f_0 = 1.355w$.
For $T_n = 0.25\,$s, $R_y = \sqrt{2\mu - 1}$, hence

$$f_y = \frac{1.355w}{\sqrt{2\mu-1}}, \qquad u_m = \frac{\mu}{\sqrt{2\mu-1}} \frac{A_y}{\omega_n^2} = \frac{\mu}{\sqrt{2\mu-1}} \frac{1.355g\,T_n^2}{4\pi^2}.$$

$$\begin{aligned}
\mu = 1: &\quad f_y = 1.355w, \ u_m = 2.104\,\text{cm},\\
\mu = 4: &\quad f_y = 0.512w, \ u_m = 3.182\,\text{cm},\\
\mu = 8: &\quad f_y = 0.350w, \ u_m = 4.347\,\text{cm}.
\end{aligned}$$