Dynamics of Structures
First Home Assignment 2013-14
due by May 20th 2014

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Instructions

This assignment is due on Tuesday May 20th. At your choice, you can either mail me your paper, in the form of a single PDF document, or hand it in *before* the class of May 20th.

For every problem, copy the text of the problem, sketch your solution, detail all the relevant passages, clearly state any required answer.

I recommend that you discuss the problems with your colleagues, but I have to remind you that

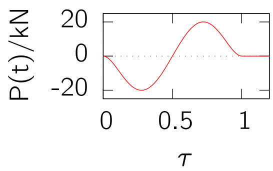
1. your paper must be the product of your individual work,
2. it's quite easy to spot *over-collaboration*.

Contents

1 Forced Response of a Damped SDOF System

A single degree of freedom, damped system has a vibration period $T = 0.250\,\mathrm{s}$, a mass $m = 1200\,\mathrm{kg}$ and a stiffness $k = 800\,000\,\mathrm{N\,m^{-1}}$. The system is at rest when it is excited by a force $p(t)$. With $\tau = t/T$ and $p_o = 560\,\mathrm{kN}$ it is

$$p(t) = p_o \begin{cases} 4\tau^5 - 10\tau^4 + 8\tau^3 - 2\tau^2 & \text{for } 0 \le \tau \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

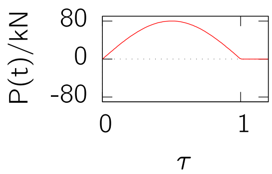


1. Find the analytical expression of the response $x(t)$ for $0 \le t \le T$.
2. Find the analytical expression of the response $x(t)$ for $T \le t$.
3. Find the peak value of the response.
4. Plot the response in the interval $0 \le t \le 3T$.

2 Numerical Integration of the EOM

The system of exercise 1 is at rest when it is subjected to a half-sine impulse. With $t_o = 0.04\,\mathrm{s}$, $\tau = t/t_o$ and $p_o = 80\,\mathrm{kN}$ it is

$$p(t) = p_o \begin{cases} \sin \pi\tau & \text{for } 0 \le \tau \le 1 \\ 0 & \text{otherwise} \end{cases}$$

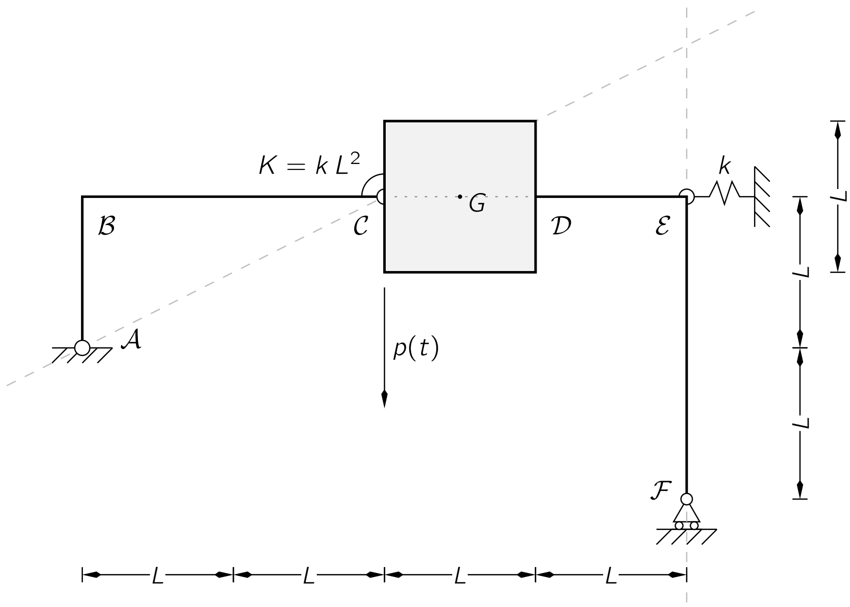


1. The duration of the excitation being short with respect to the period of the system, give an estimate of the maximum response disregarding the effects of damping.
2. Integrate numerically the equation of motion in the interval $0 \le t \le 2T$, using the linear acceleration algorithm and a time step $h = t_o/10$.
3. Plot the response in the interval $0 \le t \le 2T$.
4. Briefly discuss the choice of h. Would you recommend changing the time step in different parts of the interval of interest?

Optional

5. The spring is not indefinitely elastic, but has a yield strength $f_y = 32.5\,\mathrm{kN}$.

3 Rigid Bodies Assemblage



The system in the figure above consists of

- two rigid bodies interconnected by a hinge in $\mathcal{C}$ and connected to the system of reference by a hinge in $\mathcal{A}$ and a roller in $\mathcal{F}$,
- two springs, a rotational spring connecting the two bodies and an extensional spring connecting the rightmost rigid body to the system of reference.

The mass of the gray square is m, while the two rods are massless; a vertical load $p(t) = p_o \sin \omega t$ is applied in $\mathcal{C}$.

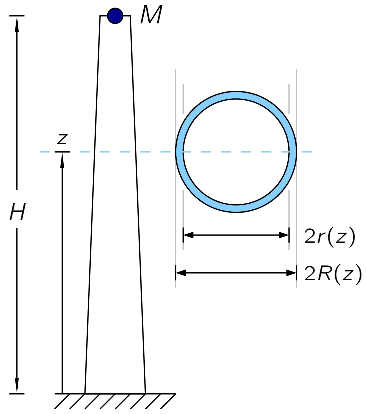
1. Using u_G (the horizontal displacement of the center of mass) as the free coordinate write the equation of motion for the system.

4 Rayleigh's Quotient, flexible system

We want to perform a preliminar study of a reinforced concrete water tower using the Rayleigh's Method.

With this purpose, the water tower is modeled as a clamped, tapered beam that supports a lumped mass, representing the tank and its content, with

- height of the beam, up to the lumped mass, $H = 45\,\text{m}$,
- lumped mass $M = 1\,200\,000\,\text{kg}$,
- mass density of concrete $\gamma = 2500\,\text{kg}\,\text{m}^{-3}$,
- Young's modulus of concrete $E = 30\,\text{GPa}$.

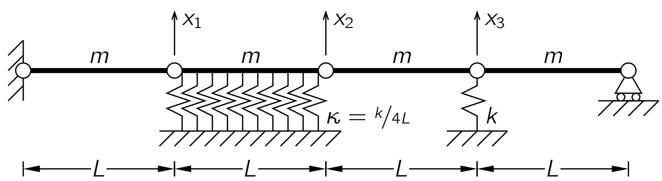


The section of the beam is annular, with outer radius that varies linearly from $R_0 = 3.20\,\text{m}$ to $R_H = 2.40\,\text{m}$ and inner radius that varies linearly from $r_0 = 2.95\,\text{m}$ to $r_H = 2.20\,\text{m}$.

1. Describe at least two possible shape functions.
2. Choose the most suitable shape function amongst the above functions.
3. Using the chosen shape function determine the fundamental frequency of vibration of the system.

For the numerical evaluation of the 3 integrals you could use the trapezoidal rule (with a sufficient number of intervals) or the library function `quad`. Eventually you could integrate analytically and substitute H and 0 in the integral...

5 Rayleigh's Quotient, rigid bodies assemblage



The 3 DOF system in figure is composed of four, identical rigid bars, of length L and mass m, connected one to the other by three internal hinges and supported by a hinge, an elastic curtain[1], a spring and a roller.

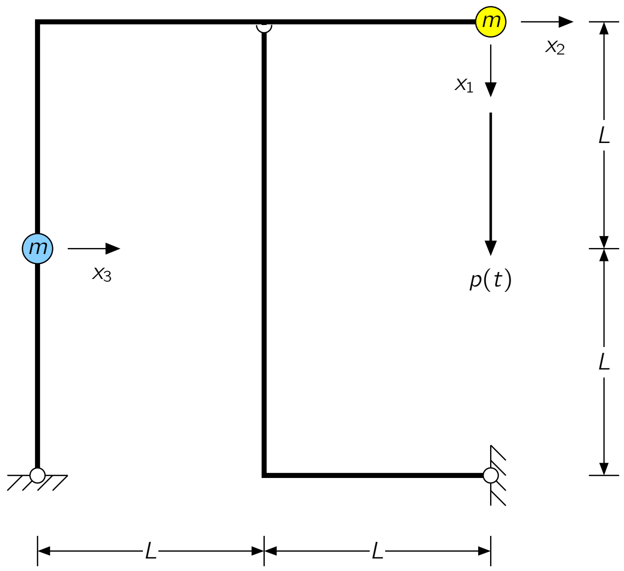
Make the assumption that the displacements are the product of a constant vector and a harmonic function,

$$\boldsymbol{u}(t) = \begin{Bmatrix} x_1 & x_2 & x_3 \end{Bmatrix}^T \sin \omega t.$$

1. Find the maximum strain energy as a function of x_1, x_2 and x_3.
2. Find the maximum kinetic energy as a function of $\dot{x}_1$, $\dot{x}_2$ and $\dot{x}_3$ (don't forget the rotatory inertias).
3. Determine the structural matrices $\boldsymbol{K}$ and $\boldsymbol{M}$.
4. Find the Rayleigh's approximations to ω^2, R_{00}, and the subsequent refinements R_{01} and R_{11}.

[1] An elastic curtain is simply a distributed elastic support, that exerts a distributed reaction force $r(s) = -\kappa\delta(s)$. Note that r is a force per unit of length, and s is a generic coordinate.

6 MDOF System, Modal Analysis, Forced Response



The system in figure is composed by two uniform beams, with constant flexural stiffness EJ, supporting two equal lumped masses, m and m.

Neglecting the mass of the beams and the axial deformability, the system can be studied as a 3 DOF system.

Using the degrees of freedom indicated in the figure, the flexibility matrix is

$$\boldsymbol{F} = \frac{1}{12}\frac{L^3}{EJ}\begin{bmatrix} 36 & -2 & -4 \\ -2 & 24 & 15 \\ -4 & 15 & 11 \end{bmatrix}.$$

1. Write down the structural matrices $\boldsymbol{M}$ and $\boldsymbol{K}$.
2. Using a method of your choice, considering the system undamped, find the eigenvalues (remember: $\omega_1^2 \le \omega_2^2 \le \omega_3^2$) and the mass-normalized eigenvectors of the system. (Express the eigenvalues in terms of $\omega_o^2 = \frac{EJ}{mL^3}$).

The system is at rest when it is excited by a vertical load $p(t)$,

$$p(t) = p_o \begin{cases} \sin(0.5\omega_o t) & \text{for } 0 \le 0.5\omega_o t \le 2\pi, \\ 0 & \text{otherwise.} \end{cases}$$

3. For each mode, write the modal equation of motion valid in the interval $0 \le 0.5\omega_o t \le 2\pi$.
4. For each mode, determine the response in terms of the sum of a homogeneous solution and a particular integral in the interval $0 \le 0.5\omega_o t \le 2\pi$.
5. For each mode, determine the free response for $2\pi \le 0.5\omega_o t$.
6. Plot the modal responses in the interval $0 \le \omega_o t \le 8\pi$.
7. Plot the displacement component $x_1(t)$ in the interval $0 \le \omega_o t \le 8\pi$.