The second 2014 Dynamics of Structures home work

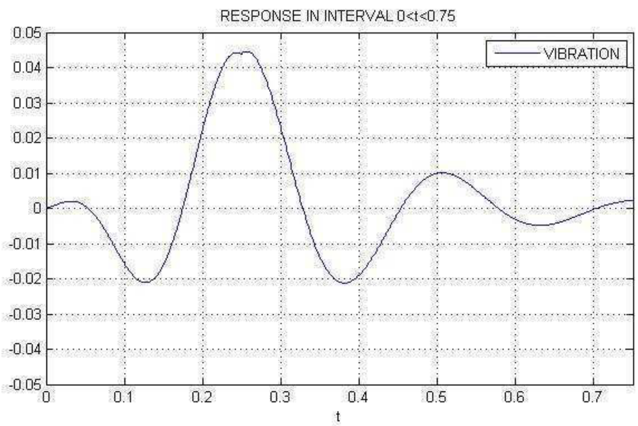
due on the day of your oral exam

Instructions

You shall present your work in the form of a paper, either printed or *nicely* hand-written, complete of all the required figures[1] and a printed listing of all the scripts/programs that you have written to derive your results.

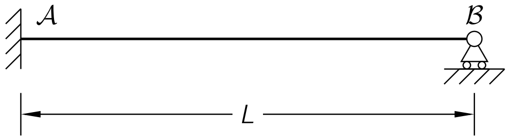
You are allowed to discuss the problems with your class mates only (or with me), but you are not allowed to work together: no common work writing the paper, no common work writing the codes.

[1]Please, do not insert bad quality images as the one on the right that, notwithstanding that it's wrong, exhibits many annoying blurry artifacts



1 Continuous System and Multiple Support Motion

The straight beam in figure is uniform, i.e., both the bending stiffness EJ and the unit mass $\bar{m}$ are constants.



The beam is at rest when it's excited by an imposed vertical motion of the support:

$$\ddot{v}_{\mathcal{B}}(t) = \omega_o^2 \delta \begin{cases} -\sin 50\,\omega_o t & \text{for } 0 \le 50\,\omega_o t \le 4\pi, \\ 0 & \text{otherwise,} \end{cases}$$

where $\delta = {}^{L}\!/_{500}$ and $\omega_o = \sqrt{{}^{EJ}\!/_{\bar{m}L^4}}$.

1. Considering only the flexural deformability and disregarding the rotatory inertia, determine for the first 3 modes

   - the wavenumbers β_i,
   - the eigenfunctions $\psi_i(x)$,
   - the modal frequencies of vibration, $\omega_i = \lambda_i \omega_o$,
   - the modal masses $M_i^\star = \mu_i \bar{m} L$,
   - the modal stiffnesses, $K_i^\star = \kappa_i {}^{EJ}\!/_{L^3}$.

   and plot the 3 eigenfunctions vs. $\xi = {}^{x}\!/_{L}$.

2. Plot the given excitation $\ddot{v}_{\mathcal{B}} = \ddot{v}_{\mathcal{B}}(t)$ in the interval $0 \le \omega_o t \le {}^{\pi}\!/_{8}$.

3. Determine the static displacements $v_s(x)$ due to a unit vertical displacement of the support in $\mathcal{B}$ and plot v_s vs. ξ.

4. For the first 3 modes:

   - write the modal equations of equilibrium, taking into account the imposed motion of the support,
   - write the integrals of the equations of motion
   - plot the normalized modal responses, ${}^{q_i}\!/_{\delta}$ in the interval $0 \le \omega_o t \le {}^{\pi}\!/_{8}$.

5. Determine the total displacement $v({}^{L}\!/_{2}, t)$ for $0 \le \omega_o t \le {}^{\pi}\!/_{8}$ approximating the dynamic response with the combined response of the first 3 modes and plot the normalized displacement, ${}^{v(L/2,\,t)}\!/_{\delta}$ in the same interval.

6. Determine the bending moment at the fixed end, $M_b(0, t)$ for $0 \le \omega_o t \le {}^{\pi}\!/_{8}$, approximating the dynamic response with the combined response of the first 3 modes and plot the normalized bending moment in the same interval.

2 Derived Ritz Vectors and Static Condensation

The finite element model of an oil rig[2] has 132 DOF and 66 dynamic DOF and is characterized by a lumped mass matrix $\boldsymbol{M}$[3] and a stiffness matrix $\boldsymbol{K}$[4].

> You can download the two matrices either by clicking on the two links in blue above or entering the addresses reported in the below footnotes in the address bar of your browser.
> After you have downloaded the structural matrices you have to read them in your software. Their format is known as *Matrix Market Format* and googling you can find how you can read the matrices in your favorite computational environment. In `Matlab`™ this can be as easy as
>
> ```
> M = mmread('bcsstm04.mtx');
> K = mmread('bcsstk04.mtx');
> ```
>
> provided that you have downloaded the M-file `mmread.m`[5] and placed it in your working directory. If you use Python, you can
>
> ```
> from scipy.io import mmread
> M = mmread('bcsstm04.mtx') ; M = M.toarray()
> K = mmread('bcsstk04.mtx') ; K = K.toarray()
> ```

1. Compute the first 5 eigenvalues[6] of the model, using a Ritz base $\boldsymbol{\Phi}^{(0)}$ composed by the first five Derived Ritz Vectors that you have computed using an initial load vector proportional to the mass distribution, $\boldsymbol{r} = a \operatorname{diag} \boldsymbol{M}$, where a is a unit acceleration.

2. Compute the first 5 eigenvalues of the model using $\boldsymbol{\Phi}^{(1)} = \boldsymbol{K}^{-1}\boldsymbol{M}\,\boldsymbol{\Phi}^{(0)}\,\boldsymbol{Z}$ as your new Ritz base, where $\boldsymbol{Z}$ is the eigenvector matrix of the initial Ritz problem.

3. Compute the 66×66 condensed matrices $\boldsymbol{M}^{\circledast}$ and $\boldsymbol{K}^{\circledast}$ using the static condensation procedure and use the condensed matrices to compute the first 5 eigenvalues using a library procedure (e.g., `eig` in `Matlab`™).

3 Earthquake Design

A SDOF system has a natural period of vibration $T_n = 0.3\,\mathrm{s}$ and its available ductility has been estimated as $\mu = 4$. For its site, for a return period of 500 years, the elastic Pseudo-Acceleration Design Spectrum has $A(0.3) = 0.7\,\mathrm{g}$.

Taking into account that $T_b \le T_n \le T_c$ (i.e., T_n lies in the range for which $A(T) = a_a \ddot{u}_{g,o}$) determine the required yield strength and the peak e-p displacement.

[2] Duff, I. S., R. G. Grimes and J. G. Lewis, Sparse Matrix Problems, ACM Trans. on Mathematical Software, vol 14, no. 1, pp 1-14, 1989.
[3] http://www.stru.polimi.it/people/boffi/dati_2014/ha02/bcsstm04.mtx.
[4] http://www.stru.polimi.it/people/boffi/dati_2014/ha02/bcsstk04.mtx.
[5] http://www.stru.polimi.it/people/boffi/dati_2014/ha02/mmread.m.
[6] Report *only* the eigenvalues, with at least 6 significant digits