New First Assignment

due on September 1st

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Instructions

Due to logistical issues you must submit this homework by email *only*, in the form of a PDF attachment produced by a word processor or a typesetting system (absolutely no scans of your handwriting, please don't strain my eyes).

For every problem, describe the procedure you're following, show the relevant intermediate results and answer the questions. You can discuss the problems with me or your classmates and no one else (please consider that my replies could be not as prompt as usual) but you must write every word, every equation, every line of code on your own.

1 Dynamical Testing

A simple structure, which can be modeled as a single degree of freedom system, is subjected to testing to measure its dynamical characteristics.

First, it is loaded with a static force $F = 30\,\text{kN}$ and the static displacement is measured: $u_0 = 15\,\text{mm}$; the force is then suddenly released and the structure is allowed to oscillate freely: after 10 cycles, corresponding to 3 s after the force release, the measured maximum displacement is $u_{10} = 10\,\text{mm}$.

What are the parameters of the SDOF system?

2 Vibration Isolation

An industrial machine has a mass $m_0 = 29\,200\,\text{kg}$ and transmits to its rigid supports a harmonic force of amplitude 2.8 kN at 25 Hz when it reaches the steady state(s-s) regime.

1. Determine the stiffness $k^{(1)}$ of an undamped, elastic suspension system for the machine, so that the amplitude of the s-s transmitted force is no greater than 700 N.

After the installation of the suspension system it is found that the forces transmitted to the supports during the transient are too large.

2. Determine $k^{(2)}$ and c such that the transmissibility ratio, TR, is still equal to 0.25 and the damping ratio of the suspension system is $\zeta = 0.10$.

Eventually, is it possible to modify the system in such a way that, using the springs that you have designed in step 1 (i.e., $k = k^{(1)}$), you still have $\text{TR} = 0.25$ and $\zeta = 0.10$?

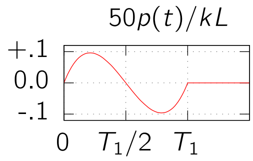
3 Numerical Integration

An undamped 2 DoF system is characterised by the structural matrices

$$\boldsymbol{M} = m\begin{bmatrix}3 & 0\\0 & 2\end{bmatrix}, \qquad \boldsymbol{K} = k\begin{bmatrix}3 & -1\\-1 & 2\end{bmatrix} \qquad (\text{with } {}^{k}\!/_{m} = \omega_0^2),$$

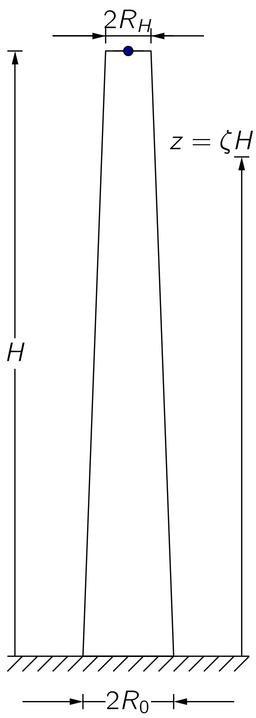
and it is at rest when it is excited by the load (NB: T_1 is the period of vibration associated with the first mode)

$$\boldsymbol{p}(t) = \begin{Bmatrix}1\\0\end{Bmatrix}\frac{kL}{50}\begin{cases}2({}^{t}\!/_{T_1})^3 - 3({}^{t}\!/_{T_1})^2 + {}^{t}\!/_{T_1} & \text{for } 0 \le t \le T_1,\\ 0 & \text{otherwise.}\end{cases}$$



1. Find the analytical expression of the response $x_1 = x_1(t)$ in the interval $0 \le t \le 3T_1$ using modal superposition.
2. Find the response of the system in the interval $0 \le t \le 3T_1$ using the linear acceleration algorithm with a time step $h \le {}^{T_2}\!/_{8}$ where T_2 is the period of vibration associated with the second mode.
3. Plot in the same figure the analytical and the numerical solutions that you have found for x_1.

4 Rayleigh Quotient



A reinforced concrete tower has to support a platform of weight $W = 14\,\mathrm{MN}$, that can be represented by a lumped mass placed at a height $H = 72\,\mathrm{m}$ above the foundation level.
The tower cross section is in the form of a circular ring with constant thickness $t = 0.25\,\mathrm{m}$, while the mean radius is a linear function of the height.

Using the Rayleigh Quotient method and the shape function $\psi = 1 - \cos{}^{\pi z}\!/_{2H}$ find the undetermined coefficients R_0 and R_H in the linear expression for the mean radius

$$R_{\text{mean}} = (1 - {}^{z}\!/_{H})\,R_0 + {}^{z}\!/_{H}\,R_H$$

in such a way that the natural period of vibration is $T_n \le 2.5\,\mathrm{s}$ and the normal stress due to pure axial load is $\sigma_N = {}^{N}\!/_{A} \le 6\,\mathrm{MPa}$ in every cross section.

For the reinforced concrete use the following values: $\rho = 2500\,\mathrm{kg\,m^{-3}}$ and $E = 30\,\mathrm{GPa}$.
Extra points to the minimum weight solution.

5 Rigid Bodies Single DoF System

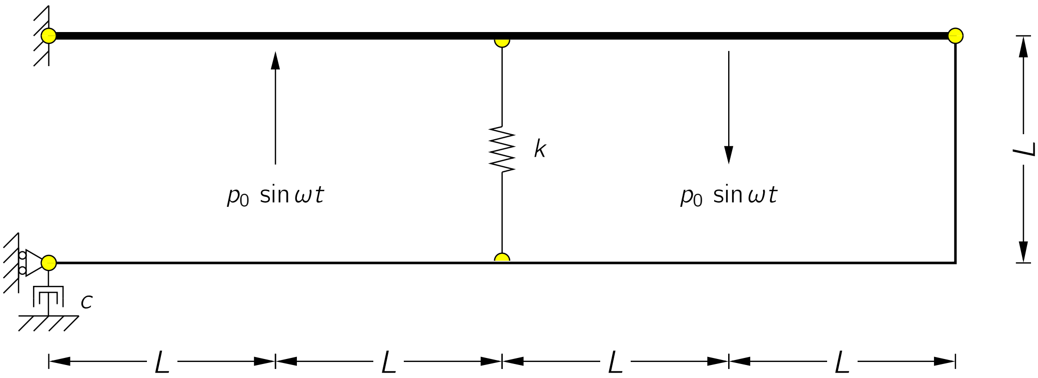


Figure 1: the rigid bodies single DoFsystem.

The single DoF system in figure 1, composed of two rigid bars connected by a hinge and an extensional spring of stiffness k, is excited by two harmonic forces, of opposite sign, both described as $p(t) = p_0 \sin \omega t$.

The massless bottom bar is connected to the system of reference by a roller and by a damper with damping constant c, the top bar is connected to the system of reference by a hinge and has a constant unit mass $\bar{m}$.

Write the equation of motion of the system

6 MDOF System

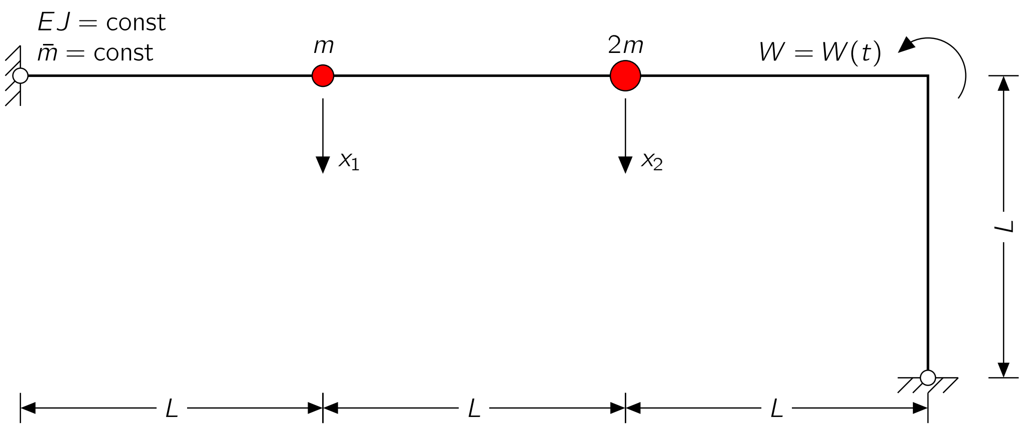


Figure 2: two DoF system.

The structure in figure 2 has the following characteristics: *a*) it is composed of a single, uniform beam hinged at both its ends, *b*) it supports two different masses (m and $2m$) in the positions indicated in figure and *c*) it is excited by a couple $W = W(t)$ applied at the top-right corner of the beam.

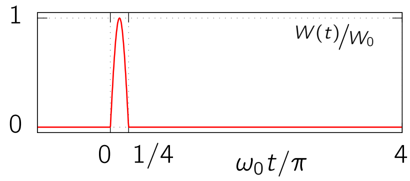
With the understanding that the total beam mass, $4\bar{m}L$ is negligible with respect to m and that the effects of the shear and of the axial deformations are negligible with respect to the effects of the flexural deformations, you can study the dynamic response of the system considering the two dynamic DoF's indicated in figure.

Remember that, to take into account the effects of W, you have to introduce a static DoF.

1. Compute the uncondensed, 3×3 stiffness matrix, using either direct assemblage or by inversion of the flexibility matrix.
2. Compute the condensed structural matrices, $\boldsymbol{K}$ and $\boldsymbol{M}$.
3. Compute the eigenvalues and the eigenvectors of the system, expressing the eigenvalues as multiples of $\omega_0^2 = {}^{EJ}/_{mL^3}$.

The couple has a maximum value $W_o = \mu {}^{EJ}/_{L}$ and is modulated by a half-sine wave:

$$W(t) = W_0 \begin{cases} sin(4\omega_0 t) & 0 \leq 4\omega_0 t \leq \pi \\ 0 & \text{otherwise} \end{cases}$$



and the system is at rest for $t = 0$.

4. Write the modal equations of motion and find the modal responses in the interval $0 \leq \omega_0 t \leq 4\pi$ in terms of analytical expressions.
5. Plot the modal responses in the same interval.
6. Plot the rotation of the top-right corner of the beam in the same interval.