Dynamics of Structures

Homework Assignment May 2015
due on Tuesday the 9th of June

Contents

Instructions

You will submit your homework by email no later than 14:30 CEST June 9.
Your submission will be composed of

- a PDF attachment named `Familyname,_Givenname.pdf`, where you will explain the procedures used to solve each problem, reporting all the relevant steps and clearly indicating the requested results, using graphical output wherever appropriate,
- a ZIP archive attachment named `Familyname,_Givenname.zip`, containing the files (e.g., Matlab scripts, Excel workbooks etc etc) that you've used preparing your answers.

The approximate value of the exercises is 4, 2, 7, 3, 6, 8 points over a total of 30 points, at least 15 points are required to be admitted to the oral exam.

You can discuss the problems with your colleagues (I actually recommend that!) or with me but I have to remind you that your homework must be the product of your individual work. Any evidence of shared results implies that this homework is not valid for your admission to the oral exam (of course you will have other opportunities later on, but after July).

1 Collision

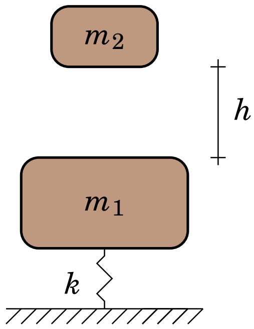
A body, its mass $m_1 = 200\,\text{kg}$, subjected to a gravitational force $g_\text{p} m_1$ is supported by an elastic spring, its stiffness $k = 125\,\text{N}\,\text{mm}^{-1}$.

The body is at rest (in equilibrium) when it is hit by another body, its mass $m_2 = 50\,\text{kg}$, falling from an height $h = 2000\,\text{mm}$.

After the collision the two bodies are *glued* together and the combined system reaches a maximum deflection $x_\text{max} = 60\,\text{mm}$.

The experiment is conducted on the surface of a planet of the Solar System, which one?

Hint: compute the value of g_p.



2 Vibration Isolation

A large machinery, its mass $m = 200\,000\,\text{kg}$, during its steady state operation transmits to its rigid supports an unbalanced harmonic load $p(t) = p_o \sin\omega_o t$, with $p_o = 1200\,\text{N}$ and $\omega_o = 2\pi\,30\,\text{rad}\,\text{s}^{-1}$.

Design two suspension systems, one undamped and the other with a damping ratio $\zeta = 6\%$, so that the maximum force transmitted to the support is reduced to $p_\text{max} = 400\,\text{N}$.

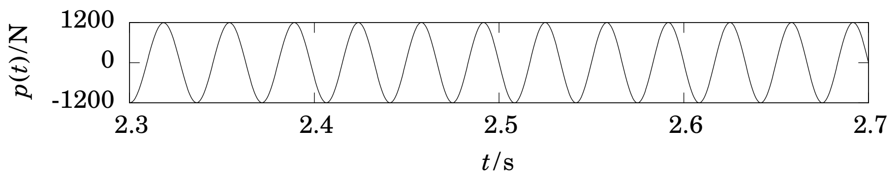
3 Numerical Integration

The machinery of the previous problem doesn't reach the steady state instantaneously and the unbalanced load has approximately the following expression:

$$p(t) = p_o \begin{cases} \dfrac{2t_o t - t^2}{t_o^2} \sin(\omega_o \dfrac{t}{2t_o} t) & 0 \le t \le t_o, \\ \sin(\omega_o(t - t_o/2)) & t_0 \le t, \end{cases}$$

with $t_o = 2.5\,\text{s}$.

1. Plot the unbalanced load in the interval $0 \le t \le 3\,\text{s}$ (e.g., here follows

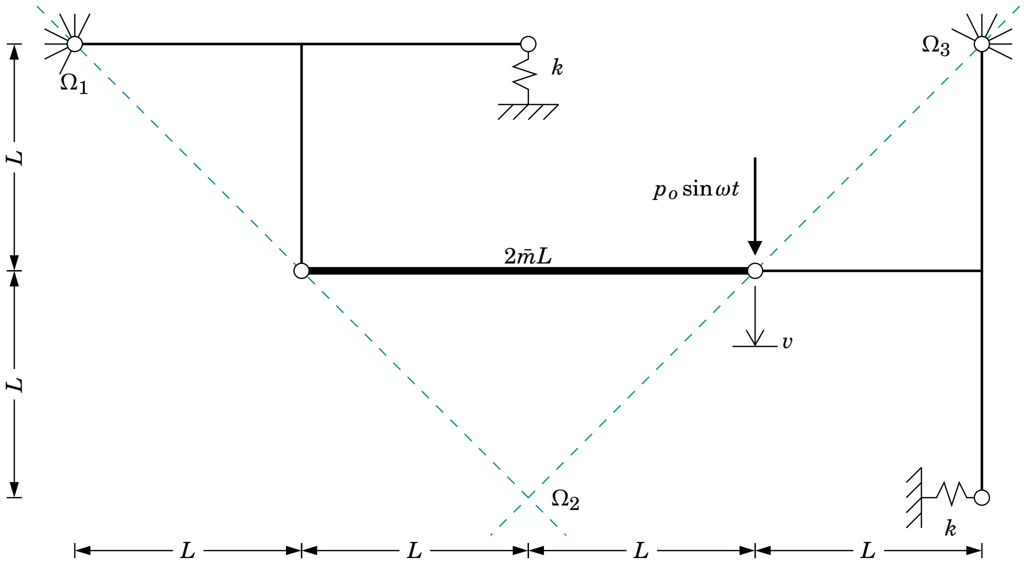


   a plot of the unbalanced load in the interval $2.4\,\text{s} \le t \le 2.7\,\text{s}$).

2. Considering the same time interval plot the force transmitted to the rigid support by the undamped suspension system during the transient, using the *linear acceleration algorithm* to determine the system response.

3. Repeat point 2 for the damped suspension system.

4 SDOF System



The SDOF system in figure is composed of 3 rigid bodies and 2 elastic springs of equal stiffness k and it is excited by a vertical force, $p(t) = p_o \sin\omega t$, applied to the right internal hinge. Only the central body has a significant mass.

Write the equation of motion using the vertical displacement of the right internal hinge as your free coordinate.

5 Rayleigh Quotient Method

The structure in figure is composed of flexible, massless columns and rigid traverses so that, neglecting the axial deformability of the columns, you can study it as a simple 3–DOF system, the DOF being the lateral displacements of the storeys.

The stiffnesses indicated in figure are to be understood as storey stiffnesses, relating the storey shear S_i to the *relative* storey displacements,

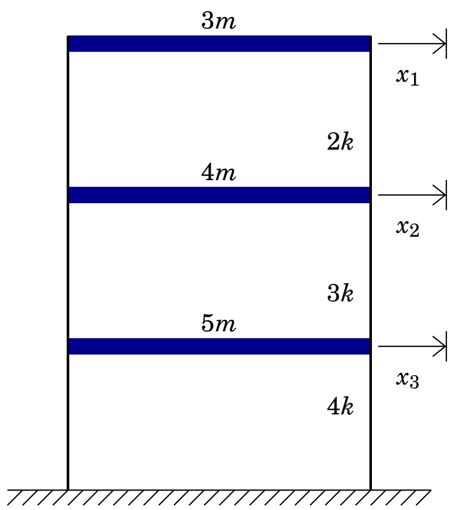
$$S_i = (x_{i+1} - x_i)k_i$$

(with the understanding that $x_4 \equiv 0$).

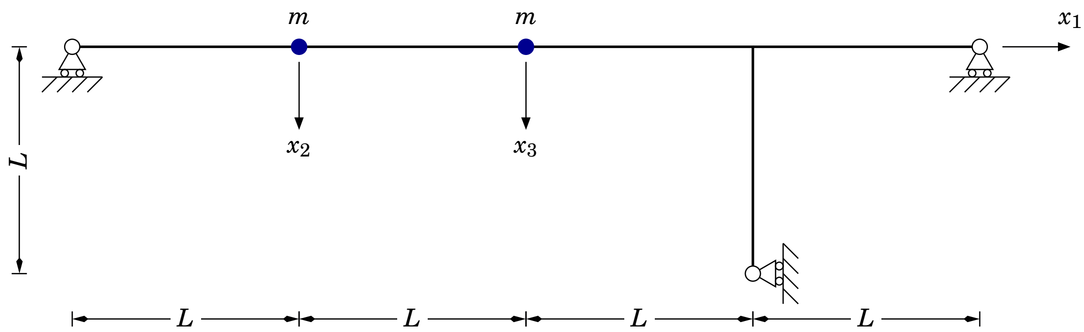
Assuming a shape vector

$$\boldsymbol{\phi}_o = \{3 \quad 2 \quad 1\}^T$$

determine the first three approximations R_{00}, R_{01} and R_{11} to the fundamental frequency of vibration using the *Rayleigh Quotient Method*.



6 MDOF System



The system in figure is composed of a single uniform beam, supporting two equal lumped masses and you can assume that

- the beam mass is negligible with respect to the supported mass,
- the damping is negligible,
- the shear and axial deformations are negligible with respect to the flexural deformations.

The system is subjected to the following initial conditions

$$\boldsymbol{x}_0 = \delta \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}, \qquad \dot{\boldsymbol{x}}_0 = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}.$$

1. Compute the system mass and stiffness matrices.
2. Compute the system eigenvalues and eigenvectors, normalized with respect to the mass matrix.
3. Write the modal equation of motion.
4. Integrate the modal equation of motion.
5. Plot the response $x_1(t)/\delta$ in the interval $0 \le t \le 5T_1$, T_1 being the period of vibration associated with the first mode.

   HINT: the direct flexibilities are

$$f_{1,1} = \frac{11}{12}\frac{L^3}{EJ}, \quad f_{2,2} = \frac{9}{12}\frac{L^3}{EJ}, \quad f_{3,3} = \frac{16}{12}\frac{L^3}{EJ}.$$