Dynamics of Structures 2014–2015

Second Home Assignment

Due on your oral exam

Contents

Instructions

The day of your oral exam you will hand in a printed copy of your solution, complete with all the requested plots, tables etc. The correction will be at the beginning of your oral exam. All the usual recommendations apply: you are free to discuss the problems with me and with your colleagues but you must work on your own.

1 Differential Support Motion

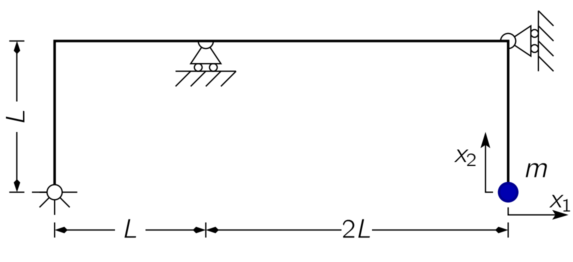


Figure 1: the model of the structure

The system in figure 1 is composed of a uniform, undamped elastic beam that supports a point mass m. The beam mass being very small with respect to the supported mass, you can study the dynamic behavior of the system considering just the two degrees of freedom indicated in figure.

Neglecting the shear and axial deformations, the flexibility matrix for the dynamical degrees of freedom can be computed using the PVD,

$$\boldsymbol{F} = \frac{L^3}{24EJ}\begin{bmatrix}63 & 62\\ 62 & 92\end{bmatrix}$$

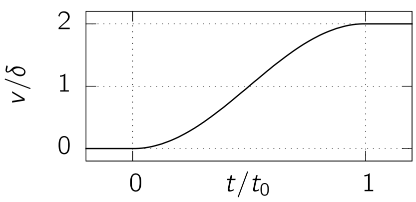
and by inversion we have the stiffness matrix,

$$\boldsymbol{K} = \frac{EJ}{244L^3}\begin{bmatrix}276 & -186\\ -186 & 189\end{bmatrix}$$

1. Compute the system eigenvalues in terms of $\omega_0^2 = {}^{EJ}\!/_{mL^3}$.
2. Compute the corresponding, mass-normalized eigenvectors.

The system is subjected to a vertical displacement of the horizontal roller,

$$v(t) = \begin{cases} 0 & t \le 0,\\ (1-\cos\pi{}^{t}\!/_{t_0})\delta & 0 \le t \le t_0,\\ 2\delta & t_0 \le t, \end{cases}$$



with $t_0 = 0.2\,T_1$, T_1 being the natural period of vibration associated with the first mode of the system.

3. Determine the influence matrix $\boldsymbol{E}$.
4. Integrate the modal equations of motion in the interval $0 \le t \le 5T_1$, considering that the system is initially at rest, and plot your results.
5. Give an analytical representation of $x_1(t)$, the horizontal displacement of the mass, in the same time interval and plot your result.
6. *Optional* Give an analytical representation of $R(t)$, the vertical reaction of the horizontal roller, in the same time interval and plot your result.

2 Structural Response

For the purpose of dynamic analysis a ten floors building is modeled as a shear-type building, as in figure 2 (note that both the storey masses and the storey stiffnesses vary along the height of the building).

1. Determine the structural matrices, $\boldsymbol{M}$ and $\boldsymbol{K}$.
2. With the help of a library function, determine the eigenvalues and the mass-normalized eigenvectors of the model.
3. In the case of a seismic excitation, determine how many modes do you need to consider in your analysis to represent, in terms of equivalent static forces, the base shear and the overturning moment with an error smaller than 3%.

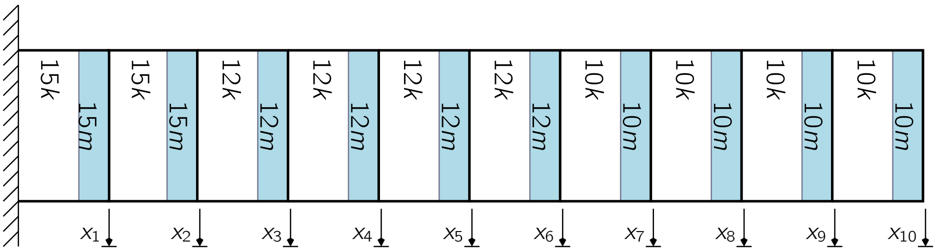


Figure 2: model of the building, rotated by $-90°$.

3 Matrix Iteration

With reference to the building model of exercise 2 answer the following points and comment, briefly comment, your findings.

1. Using $\boldsymbol{\phi}_0 = \begin{Bmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10\end{Bmatrix}^T$ compute the Rayleigh Quotient approximations R_{00}, R_{01} and R_{11} to the first eigenvalue of the system.

2. Using the Ritz base

$$\boldsymbol{\Phi}_0 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 5 & 4 & 2 & 0 & -3 \\ 1 & 2 & 3 & 2 & 0 & -2 & -2 & 0 & 1 & 3 \\ 1 & 1 & -1 & -1 & 0 & 2 & 3 & 1 & -2 & -6 \\ 1 & 1 & -1 & -1 & 2 & 2 & -3 & -3 & 2 & 6 \end{bmatrix}$$

   compute an approximation to the first five eigenvalues and eigenvectors of the model using the Ritz-Rayleigh method.

3. Using the previous results, perform one subspace iteration to compute a possibly better approximation to the first five eigenvalues and eigenvectors of the model.

4 Continuous System

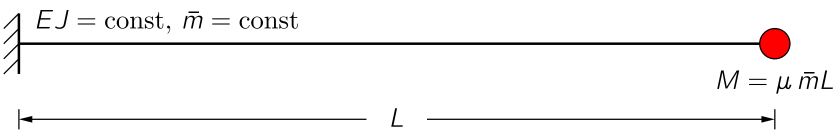


Figure 3: the continuous system.

The system in figure 3 is composed of a uniform, undamped elastic beam that is clamped at the left and supports a mass at the right end. For this problem you can neglect the effects of shear deformations and rotatory inertia.

1. Compute numerically the first eigenvalue of the system as a function of the supported mass,
$$\omega_1^2 = \omega_1^2(\mu), \text{ for } 0 \le \mu \le 10$$
and plot your results.

2. For $\mu = 6$, compute the first five wavenumbers, eigenvalues and mass normalized eigenfunctions.

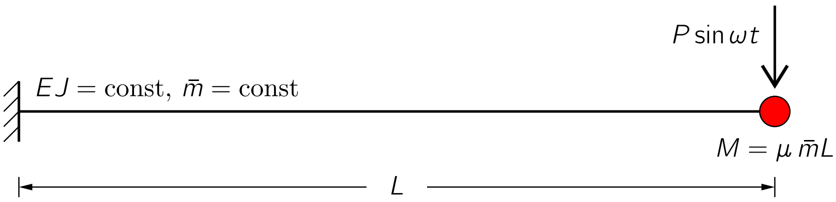


Figure 4: loaded system

The system is loaded by a concentrated load as in figure 4,

$$P(t) = \frac{EJ}{500\,L^2} \begin{cases} 0 & t \le 0, \\ \sin \omega t & 0 \le t, \end{cases} \qquad \text{(where } \omega = 8\omega_0 \text{ with } \omega_0^2 = \frac{EJ}{\bar{m}L^4}\text{)}$$

3. For $\mu = 6$, compute the response $v(L, t)$ for $0 \le t \le 5T_1$, T_1 being the natural period of vibration associated with the first mode, taking into account the first five modal responses.

4. *Optional.* Compute the response $V(0, t)$ (base shear) for $0 \le t \le 5T_1$, taking into account the first five modal responses.