In the Summertime Homework

Dynamics of Structures 2016

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Instructions

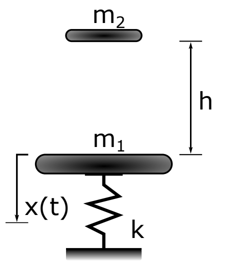
This homework consists of six problems, their relative weight being 3, 3, 6, 3, 3, 12. For each problem describe briefly your procedure and give all the required answers, detailing all the relevant steps that led to your results.

You can discuss the problems with your colleagues, with me, with other Faculty members and no one else. Every other form of collaboration is explicitly prohibited and will imply the invalidation of your homework when detected.

You will submit this homework by email, not after Friday September 4th, as a `PDF` attachment[1] named `Firstname_Lastname.pdf`.

If you want to submit your program files, your spreadsheets etc, place them in a `ZIP` archive[2,3] attachment named `Firstname_Lastname.zip`.

1 Impact



A body is in equilibrium under its own weight, $w_1 = m_1 g$ (with $m_1 = 75\,\mathrm{kg}$, $g = 10\,\mathrm{m\,s^{-2}}$) and the reaction of an elastic support of stiffness $k = 0.4\,\mathrm{kN\,mm^{-1}}$.

At time $t = 0$, with $x(0) = 0$ and $\dot{x}(0) = 0$, the body is hit by a second body, its mass $m_2 = m_1/3$, falling from a height $h = 1400\,\mathrm{mm}$. After the impact the two bodies are *glued* together (anelastic impact).

Which is the value of the maximum displacement? At what time is the maximum displacement reached for the first time?

[1] A `PDF` as produced by *Word*, TeX etc. No scans of your handwriting, thank you.

[2] A `ZIP` archive, please, not a different type of archive.

[3] Do not put the `PDF` inside the archive!

2 Vibration Isolation

An industrial machine has a mass $m_0 = 7100\,\mathrm{kg}$ and transmits to its rigid supports a harmonic force of amplitude $2.4\,\mathrm{kN}$ at $20\,\mathrm{Hz}$ when it reaches the steady state regime.

1. Determine the stiffness k_{susp} of an undamped, elastic suspension system for the machine, so that the amplitude of the s-s transmitted force is no greater than $300\,\mathrm{N}$.

After the installation of the suspension system it is found that the forces transmitted to the supports during the transient are too large.

2. Modify the dynamical system parameters (i.e., m, c and k) so that
   (a) the damping ratio of the new suspension system is $\zeta = 8\%$,
   (b) the stiffness doesn't change, $k = k_{\mathrm{susp}}$, because you want to use the same springs that you have already installed and
   (c) the transmissibility ratio doesn't increase.

3 Numerical Integration

A damped dynamical SDOF system characterized by the following parameters,

$$m = 12\,\mathrm{kg}, \qquad \zeta = 3.8\%, \qquad k = 13.2\,\mathrm{kN\,m^{-1}},$$

is at rest when it is excited by the load

$$p(t) = 1\,\mathrm{kN} \times \begin{cases} \tau^3 - 1.4\tau^2 + 0.44\tau + 0.4 & \text{for } 0 \le t \le t_1 \\ 0 & \text{otherwise} \end{cases}$$

with $t_1 = 1.2\,\mathrm{s}$ and $\tau = \frac{t}{t_1}$.

1. Plot the load in the interval $-1\,\mathrm{s} \le t \le 2\,\mathrm{s}$.

2. Give the analytical expression of the system displacement in the interval $0 \le t \le 2\,\mathrm{s}$.

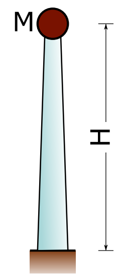
3. Integrate numerically, using the linear acceleration method and a time step $h = T_n/12$, the equation of motion in the interval $0 \le t \le 2\,\mathrm{s}$ (T_n being the undamped period of vibration of the system).

4. Plot the analytical response and the numerical approximation in the interval $0 \le t \le 2\,\mathrm{s}$.

4 Rayleigh Quotient Method

You want to study the natural frequency of vibration of a tapered beam supporting a lumped mass.

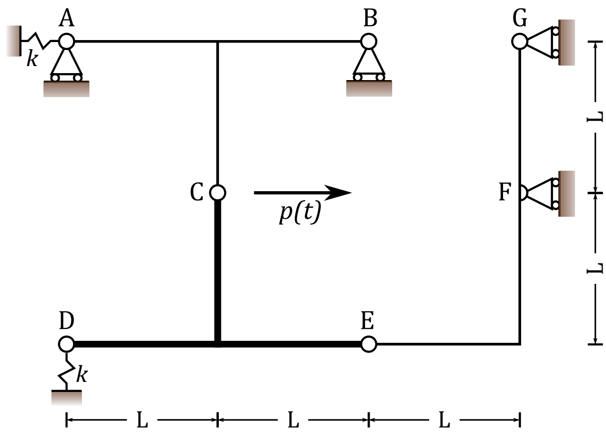
The beam is clamped at $z = 0$ and has a free end at $z = H = 32\,\mathrm{m}$, where it supports a lumped mass $M = 80\,000\,\mathrm{kg}$. The beam characteristics are as follows:



- the material is reinforced concrete, with a specific mass $\rho = 2500\,\mathrm{kg\,m^{-3}}$ and a Young modulus $E = 30\,\mathrm{GPa}$,
- the section is annular, with constant thickness $t = 250\,\mathrm{mm}$ and outer radius $R_e(z) = 1800\,\mathrm{mm} - \frac{z}{H}\,600\,\mathrm{mm}$.

Using two different shape functions of your choice compute two approximate values of ω^2 (the eigenvalue associated with the first mode of vibration of the system) using the Rayleigh Quotient Method and the trapezoidal rule of integration with $\Delta z = H/8$.

5 Rigid System



The system in figure is composed of three rigid bars, two elastic springs and four rollers and it is loaded by a time varying force $p(t)$,

The bars ABC and EFG are massless while the bar CDE has a unit mass $\bar{m}$ and hence its total mass is $3\bar{m}L$.

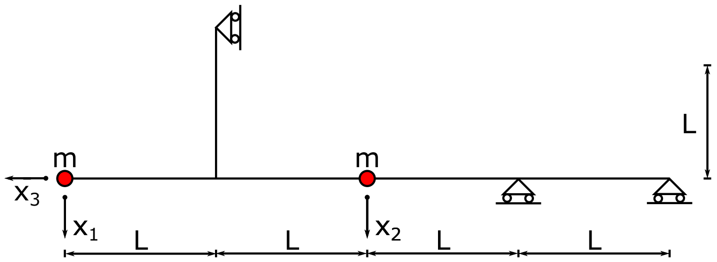
The two springs, in A and in D, have the same stiffness k.

Using the Principle of Virtual Displacements for a rigid system write the equation of motion, using as the free coordinate the horizontal displacement of the internal hinge C.

HINT: you can write the virtual work of the inertial forces acting on the bar CDE as the sum of the virtual works of the inertial forces acting on each of the three rectilinear sub-bars that compose CDE.

6 3 DOF System

The undamped system in figure is composed of a single elastic, uniform beam that supports two equal lumped masses and it is constrained by three rollers.



Considering that the mass of the beam is negligible with respect to the lumped masses and that the axial and shear deformations are negligible with respect to the flexural ones, you can use a 3 DOF system to investigate the dynamic behaviour of the system.

1. Determine the structural matrices $\boldsymbol{M}$ and $\boldsymbol{K}$.
2. Determine the eigenvalues $\omega_i^2 = \Lambda_i^2 \omega_0^2$ (with $\omega_0^2 = EJ/L^3$) and the mass-normalized eigenvectors $\boldsymbol{\psi}_i$ (mass-normalized means that $\boldsymbol{\psi}_i^T \boldsymbol{M} \boldsymbol{\psi}_i = m$).

The system is at rest when it is loaded by a time varying load,

$$\boldsymbol{p}(t) = 2\delta \frac{EJ}{L^3} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \begin{cases} \sin^2\left(\dfrac{\omega_0 t}{2}\right) & 0 \le t \le \frac{2\pi}{\omega_0} \\ 0 & \text{otherwise.} \end{cases}$$

3. Give the analytical expressions of the modal responses in the interval $0 \le t \le 4\pi/\omega_0$ and plot them.
4. Plot the nodal displacement x_1 in the interval $0 \le t \le 4\pi/\omega_0$.
5. Compute the nodal displacements and the nodal velocities, all of them, at time $t = 4\pi/\omega_0$.

   HINT: $2\sin^2(\omega t) = 1 - \cos(2\omega t)$