Dynamics of Structures 2009-2010

Last home assignment, due by Sunday, February 7th 2016

Instructions

This homework, due by Sunday, February 7th 2016 in the form of an email containing a PDF attachment, is the only HW you need to submit for the February exam.

For each problem *(a)* copy the text of the problem, *(b)* summarize the procedure you'll be using, *(c)* write down all the relevant steps (showing parts of the intermediate numerical results as you see fit), *(d)* clearly state the required answers.

If you want to submit spreadsheets, program sources etc, please submit them as separate, uncompressed attachments in the same email.

The maximum scores for each one of the five problems are respectively (and approximately) 10, 25, 15, 30, 20 (100 points total). 60 points correspond of course to a barely sufficient 18/30 score, but you need to score just 50 points to be admitted to the oral exam.

You can (and, in my opinion, you should) discuss the problems with your colleagues, with me or with other members of the teaching staff but your paper must be strictly the result of your individual effort, otherwise you will not be admitted to the examination.

1 Vibration Isolation

A rotating machine is characterized by

— its mass, $m = 108\,000\,\mathrm{kg}$;

— its working frequency, $f_\mathrm{w} = 30\,\mathrm{Hz}$,

— the value of the unbalanced load it exerts on its supports, $f_\mathrm{w} = 4200\,\mathrm{N}$.

Design a suspension system for the machine knowing that it is necessary to reduce the transmitted force to $1000\,\mathrm{N}$ and that, to reduce the vibration amplitude during transients, the suspension must have a viscous damping ratio of 7%.

2 Numerical Integration

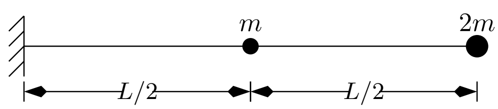


Figure 1: 2 DOF system

The structure in figure 1 is composed of a straight, uniform beam in reinforced concrete, $E = 30\,\mathrm{GPa}$ (the section of the beam is annular, with an external radius $r = 1.20\,\mathrm{m}$ and an inner radius $r = 0.90\,\mathrm{m}$) and two supported rigid bodies of different mass. The lenght of the beam is $L = 45\,\mathrm{m}$, the reference mass is $m = 300 \times 10^3\,\mathrm{kg}$.

Assuming that the mass of the beam is small with respect to the mass of the supported bodies, study the system as a 2 DOF system.

1. Compute the structural matrices, $\boldsymbol{K}$ and $\boldsymbol{M}$.

2. Compute the eigenvalues and the mass-normalized eigenvectors of the structural model.

3. Using the linear acceleration method and a time step $h = T_2/6$ (where T_2 is the period of vibration associated with the second mode) compute the displacements of the larger mass when the system is subjected to a force, applied to the smaller mass, $p(t) = 80\,\mathrm{kN} \sin \omega t,\ 0 \leq t \leq 2\pi\,\mathrm{s}$ with $\omega = 1\,\frac{\mathrm{rad}}{\mathrm{s}}$ (the system starts from rest conditions).

4. Plot the displacements of the larger mass in the time interval $0 \leq t \leq 4\pi\,\mathrm{s}$.

3 Rayleigh's Quotient

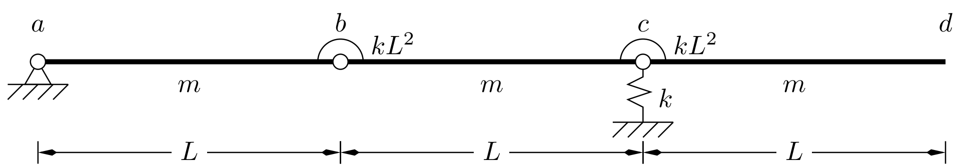


Figure 2: rigid bars and springs

The system in figure 2 is composed of three equal rigid bars, a spring, two flexural springs and a fixed support; the bars are connected by two internal hinges.

Using the vertical displacements of the points b, c and d as your degrees of freedom,

1. determine the structural matrices of the system,
2. estimate the natural frequency of vibration using the Rayleigh quotient method,
3. find the first two refinements R_{01} and R_{11} of the initial Rayleigh quotient estimate R_{00}.

4 Multiple *DOF* System

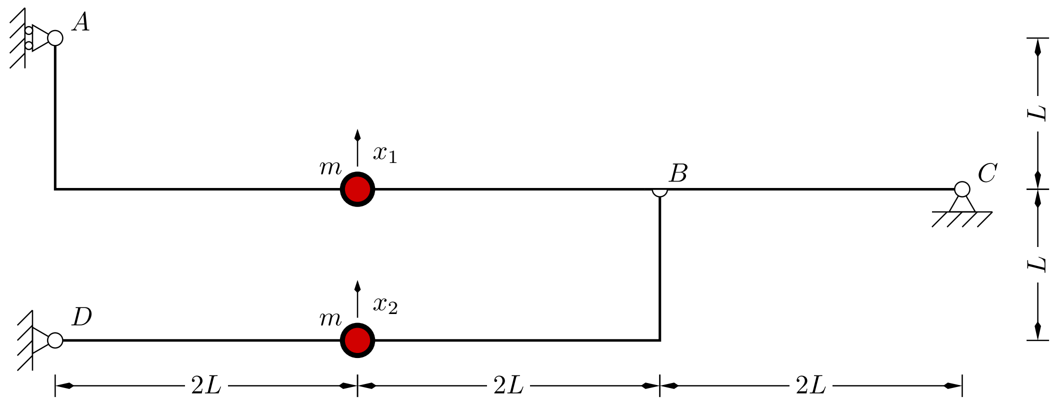


Figure 3: MDOF system

The system in figure 3, composed of two uniform, massless beams with the same flexural stiffness EJ and two supported rigid bodies with equal mass m, is at rest when the support in C is subjected to a vertical, upwards motion

$$v_C = \frac{L}{200} \times \begin{cases} 0 & \text{for } \tau \leq 0, \\ 640\tau^3 - 3840\tau^4 + 6144\tau^5 & \text{for } 0 \leq \tau \leq \frac{1}{4}, \\ 1 & \text{for } \frac{1}{4} \leq \tau. \end{cases}$$

with $\tau = t/T_1$, T_1 being the natural period of vibration of the first mode of the structure.

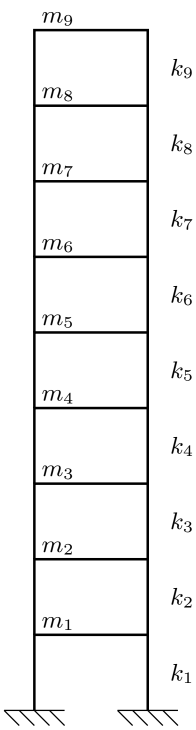
1. Write the structural matrices, find the eigenvalues solving the determinantal equation, find the mass-normalized eigenvectors of the system.

2. Determine the influence matrix for the vertical motion of the support in C.

3. Write the modal equation of motion and determine analitically the modal responses in the interval $0 \leq \tau \leq 1$.

4. Plot the deflection x_1 and the total displacement $x_{1,\text{tot}}$ in the same time interval.

5 Rayleigh-Ritz, Subspace Iteration, Derived Ritz Vectors



The structure on the left can be analyzed as a shear type building. Individual storey masses are given by

$$m_n = 405 \times 10^3\,\mathrm{kg} - n \times 5 \times 10^3\,\mathrm{kg}$$

(in tonnes), while individual storey stiffnesses are

$$k_n = 840.0\,\frac{\mathrm{MN}}{\mathrm{m}} - (n-1)^2 \times 10\,\frac{\mathrm{MN}}{\mathrm{m}},$$

(N.B.: individual storey stiffnesses k_n differ from k_{nn}, i.e., the corresponding diagonal elements of $\boldsymbol{K}$).

1. Determine the structural matrices $\boldsymbol{K}$ and $\boldsymbol{M}$.

2. Choose a 9×3 Ritz base $\hat{\boldsymbol{\Phi}}_0$, satisfying these conditions: first base vector, no changes of sign; second one, one change of sign; third one, two changes of sign.

3. Estimate the lowest three eigenvalues and eigenvectors of the structure (the eigenvectors are 9×1 vectors) using the Rayleigh-Ritz procedure, using the Ritz base of the previous step and denoting the eigenvector matrix in Ritz coordinates with $\boldsymbol{Z}_0$.

4. Perform one subspace iteration, deriving a new set of Ritz base vectors, $\hat{\boldsymbol{\Phi}}_1 = \boldsymbol{K}^{-1}\boldsymbol{M}\boldsymbol{\Phi}\boldsymbol{Z}_0$ and computing a new estimate of the lowest three eigenvalues and eigenvectors of the structure

5. Compute, using a library function, the lowest three eigenvalues and eigenvectors of the structure and compare with your previous estimates.