Constant_Acceleration

April 4, 2016

```
In [1]: %matplotlib inline
        import numpy as np
        import matplotlib.pyplot as plt
        from numpy import cos, exp, pi, sin, sqrt
```

1 Constant Acceleration

1.1 Define the Dynamical System

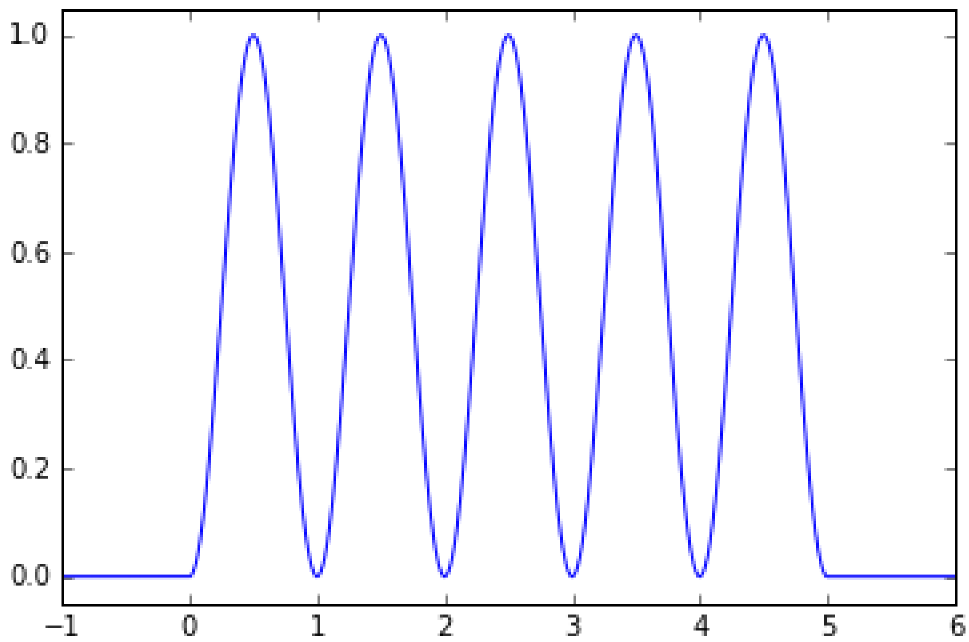
```
In [2]: m = 1.00
        k = 4*pi*pi
        wn = 2*pi
        T = 1.0
        z = 0.02
        wd = wn*sqrt(1-z*z)
        c = 2*z*wn*m
```

1.1.1 Define the Loading

Our load is

$$p(t) = 1\text{N} \times \begin{cases} \sin^2 \frac{\omega_n}{2} t & 0 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$$

```
In [3]: NSTEPS = 200 # steps per second
        h = 1.0 / NSTEPS

        def load(t):
            return np.where(t<0, 0, np.where(t<5, sin(0.5*wn*t)**2, 0))
        t = np.linspace(-1, 6, 7*NSTEPS+1)
        plt.plot(t, load(t))
        plt.ylim((-0.05, 1.05));
```



1.1.2 Numerical Constants

We want to compute, for each step

$$\Delta x = \frac{\Delta p + a^\star a_0 + v^\star v_0}{k^\star}$$

where we see the constants

$$a^\star = 2m, \quad v^\star = 2c + \frac{4m}{h},$$

$$k^\star = k + \frac{2c}{h} + \frac{4m}{h^2}.$$

```
In [4]: kstar = k + 2*c/h + 4*m/h/h
        astar = 2*m
        vstar = 2*c + 4*m/h
```

1.1.3 Vectorize the time and the load

We want to compute the response up to 8 seconds

```
In [5]: t  = np.linspace(0, 8+h, NSTEPS*8+2)
        P = load(t)
        DP = P[+1:]-P[:-1]
```

1.2 Integration

1. Prepare containers
2. write initial conditions
3. loop on load and load increments
4. vectorize reults

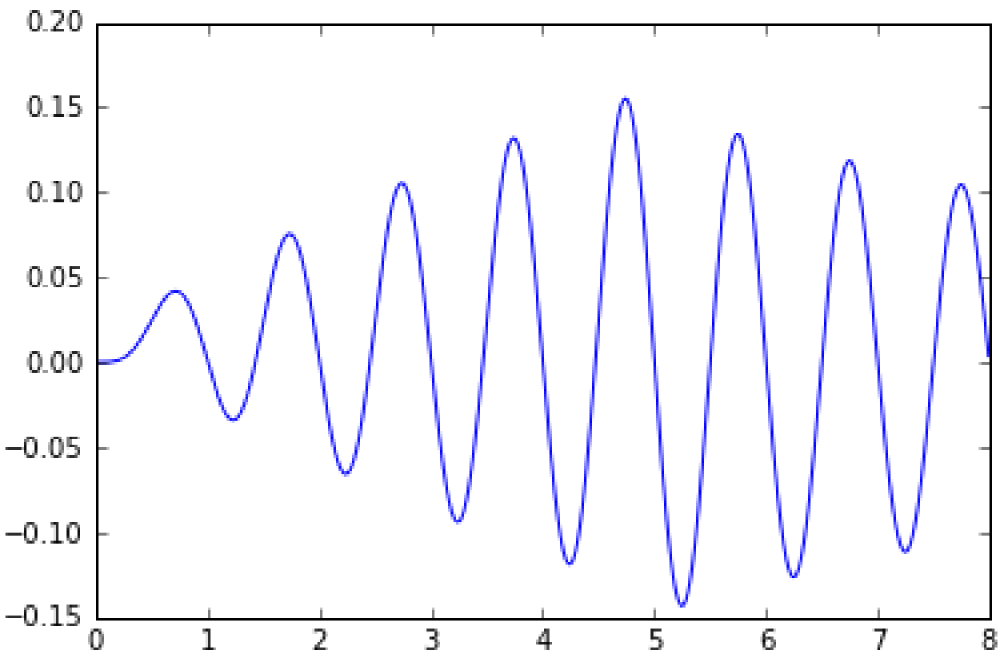
In [6]:
```
x, v, a = [], [], []
x0, v0 = 0.0, 0.0

for p, dp in zip(P, DP):
    a0 = (p - k*x0 - c*v0)/m
    x.append(x0), v.append(v0), a.append(a0)
    dx = (dp + astar*a0 + vstar*v0)/kstar
    dv = 2*(dx/h-v0)
    x0, v0 = x0+dx, v0+dv

x, v = np.array(x), np.array(v)
```

1.3 Results

1.3.1 The response

In [7]:
```
plt.plot(t[:-1],x)
```

Out[7]:
```
[<matplotlib.lines.Line2D at 0x7f3baaaa5d68>]
```



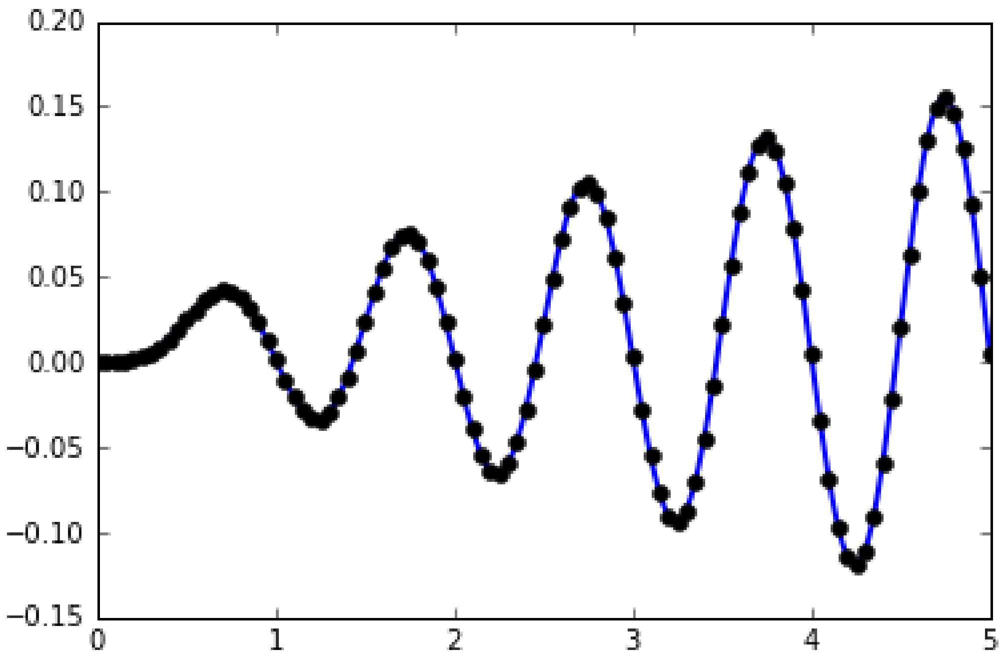
1.3.2 Comparison

Using black magic it is possible to divine the analytical expression of the response during the forced phase,

$$x(t) = \frac{1}{2k}\left(\left(\frac{1-2\zeta^2}{2\zeta\sqrt{1-\zeta^2}}\sin\omega_D t - \cos\omega_D t\right)\exp(-\zeta\omega_n t) + 1 - \frac{1}{2\zeta}\sin\omega_n t\right), \qquad 0 \le t \le 5.$$

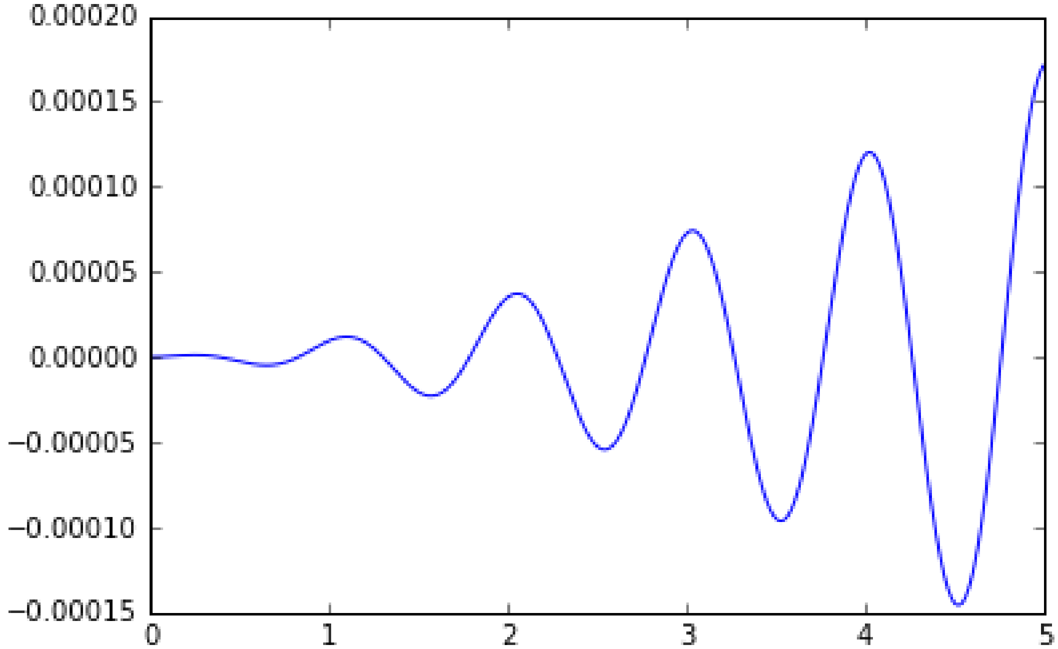
and hence plot a comparison within the exact response and the (downsampled) numerical approximation, in the range of validity of the exact response.

```
In [8]: xe = (((1 - 2*z**2)*sin(wd*t) / (2*z*sqrt(1-z*z)) - cos(wd*t)) *exp(-z*wn*t) + 1. - sin(wn*t)/(2

        plt.plot(t[:1001], xe[:1001], lw=2)
        plt.plot(t[:1001:10], x[:1001:10], 'ko')

Out[8]: [<matplotlib.lines.Line2D at 0x7f3baaadaa20>]
```



Eventually we plot the difference between exact and approximate response,

```
In [9]: plt.plot(t[:1001], x[:1001]-xe[:1001])

Out[9]: [<matplotlib.lines.Line2D at 0x7f3baa9fc8d0>]
```


0.00020
0.00015
0.00010
0.00005
0.00000
−0.00005
−0.00010
−0.00015
0
1
2
3
4
5