Numerical Integration for Non Linear Systems

Step-by-step Numerical Procedures

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Non Linear Systems

A convenient procedure for integrating the response of a non linear system is based on the incremental formulation of the equation of motion, where for the stiffness and the damping were taken values representative of their variation during the time step: in line of principle, the mean values of stiffness and damping during the time step, or, as this is usually not possible, their initial values, k_0 and c_0. The Newton-Raphson method can be used to reduce the unbalanced forces at the end of the step.

Non Linear Systems

Usually we use the modified Newton-Raphson method, characterised by not updating the system stiffness at each iteration. In pseudo-code, referring for example to the Newmark Beta Method

```
k_ini = tangent_stiffness(...)
kTilde = k0-km*m-kc*c
x1,v1,f1 = x0,v0,f0
Dr = DpTilde
loop:
   Dx = Dr/kTilde
   x2 = x1 + Dx
   v2 = gb*Dx/h + (1-gb)*v1 + (1-gb/2)*h*a0
   x_pl = update_u_pl(...)
   f2 = k*(x2-x_pl)
   % important
   Df = (f2-f1) + (kTilde-k_ini)*Dx
   Dr = Dr - Df
   x1, v1, f1 = x2, v2, f2
   if ( tol(...) < req_tol ) BREAK loop
```

Exercise

A system has a mass $m = 1000\text{kg}$, a stiffness $k = 40000\text{N/m}$ and a viscous damping whose ratio to the critical damping is $\zeta = 0.03$.
The spring is elastoplastic, with a yielding force of 2500N.
The load is an half-sine impulse, with duration 0.3s and maximum value of 6000N.
Use the constant acceleration method to integrate the response, with $h = 0.05\text{s}$ and, successively, $h = 0.02\text{s}$. Note that the stiffness is either 0 or k, write down the expression for the effective stiffness and loading in the incremental formulation, write a spreadsheet or a program to make the computations.