June 2016 Homework

Dynamics of Structures 2015-2016

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Instructions

This assignment, required for admission to the oral exams of July, is due not later than Wednesday June 30th. You can score 9, 4, 17 points for problems 1, 2, 3 respectively, you have to score more than 17 points for admission.

Submit your work by email, [1] in the form of a PDF attachment [2] containing your solutions: for every problem describe your procedure with sufficient detail and report the intermediate results necessary to derive the required answers.

If you desire to submit further materials (e.g., source code, spreadsheets, endless lists of data produced by your programs etc) provide them as *separate* attachments. Please don't send me an archive (.ZIP or .RAR) with all your files within. Please. Don't.

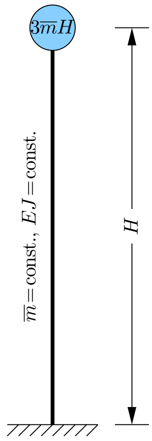
You are allowed, or rather encouraged to discuss the problems with your colleagues, but you shall not discuss the problems with anyone else, except for me or other members of the Faculty.

By the way, *to discuss a problem is* absolutely different *from sharing parts of its solution* and any evidence of sharing will mean no admission to July exams, no admission for all the involved parties.

[1] Address your email to `giacomo.boffi@polimi.it`

[2] Prepare your PDF using a word processor or a typesetting program, do not prepare it sticking together photos of a manuscript.

1 Rayleigh Quotient



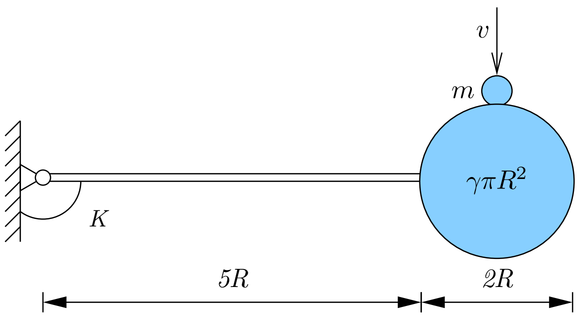
A tower structure is composed of a straight, uniform beam clamped at one end and supporting a massive body at the other end.
The beam is characterized by its length H, its unit mass $\overline{m}$ and its flexural stiffness EJ, the supported body is characterized by its mass $m{=}3\overline{m}H$.

1. Using the shape function $\phi(x) = ({}^{x}/_{H})^2$ compute the Rayleigh Quotient estimate of the first eigenvalue of the structure, R_{00}.
2. Compute the first refinement of the Rayleigh Quotient estimate, R_{01}.
3. Compute the second refinement, R_{11}.

1.1 Notes

1. In your derivations neglect the shear and axial deformations of the beam, as well as any form of rotatory inertia.
2. You will have to compute quite a number of integrals... Some of them must be computed analytically (e.g., $|M(x)|{=}|\int_x^H V(s)\mathrm{d}s|$), others can be computed either analytically (some are *complicated* but none is complex) or using a numerical procedure (say composite Simpson rule, etc).

2 Impact



A rigid body is composed of a circle, its radius R and its mass $\gamma\pi R^2$ and a rectilinear rod, its length $5R$ and its mass negligible with respect to circle mass.

The rod, perpendicular to the circle, is rigidly connected to it and it is hinged at the other end. A further constraint, applied to the hinged end, is a flexural spring of stiffness K.

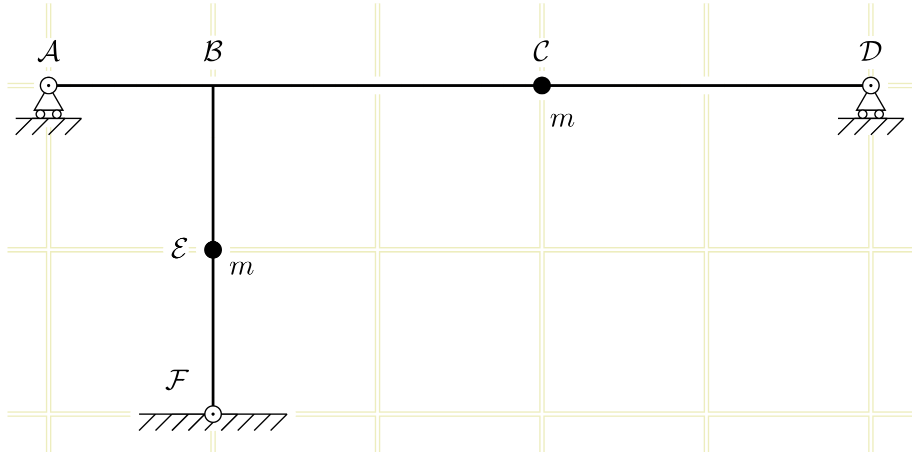
The frequency of vibration of the system, measured experimentally for rotations of small amplitude ($\theta{\approx}\sin\theta$), is equal to ω_n.

The system is at rest when it's hit by a second body moving in the direction of the centre of the circle and whose velocity is perpendicular to the rod. The dimensions of the moving body are negligible and the impact is inelastic: after the impact the bodies are *glued* together.

The amplitude and the frequency of the oscillatory motion of the two bodies after the impact were measured, $\theta_{\max}=\theta_0$ and $\omega=\alpha\omega_n$, $0<\alpha<1$.

Find the mass and the velocity of the impacting body in the hypothesis that the ensuing motion is of small amplitude.

3 Support Motion



The structure in figure supports two equal masses in $\mathcal{C}$ and $\mathcal{E}$ and is composed of two uniform beams, both with flexural stiffness EJ, namely

- a horizontal beam $\overline{\mathcal{ABCD}}$ of length $5L$ and
- a vertical beam $\overline{\mathcal{BEF}}$ of length $2L$,

that are rigidly connected in $\mathcal{B}$.

The mass of the beams is negligible with respect to the supported masses and the axial and shear deformabilities of the beams are negligible with respect to their flexural deformability.

1. The structure is at rest when it's subjected to a horizontal ground acceleration,

$$\ddot{u}_g(t)=\delta\omega_0^2\begin{cases}\sin\omega_0 t & 0\le\omega_0 t\le 2\pi,\\ 0 & \text{otherwise}\end{cases}$$

   where δ is a length and $\omega_0=\sqrt{EJ/mL^3}$ is a reference frequency.

   (a) Find the analytic expressions of modal responses for $0\le\omega_0 t\le 4\pi$.

   (b) Using the above analytic expressions, compute and plot $v_{\mathcal{C}}(t)$, the vertical displacement of the mass in $\mathcal{C}$, in the same time interval.

   (c) Verify your solution for $v_{\mathcal{C}}$ against a numerical solution, computed using the constant acceleration method.

2. The structure is at rest when it is subjected to a vertical motion of the support in $\mathcal{D}$,

$$v_{\mathcal{D}}(t)=\delta\begin{cases}0 & \omega_0 t<0,\\ \omega_0 t-\sin\omega_0 t & 0\le\omega_0 t\le 2\pi,\\ 2\pi\omega_0 & 2\pi<\omega_0 t.\end{cases}$$

(a) Plot $v_{\mathcal{D}}(t)$, $\dot{v}_{\mathcal{D}}(t)$ and $\ddot{v}_{\mathcal{D}}(t)$ in the interval $0 \le \omega_0 t \le 4\pi$.

(b) Compute and plot the total vertical displacement of the mass in $\mathcal{C}$, $v_{\mathcal{C}}(t)$ in the same time interval.

3.1 Notes

Dynamic Degrees of Freedom The mass of the beams being negligible with respect to the supported masses, the axial and shear deformabilities of the beams being negligible with respect to their flexural deformability, it is possible to model the dynamic behavior of the structure using three dynamical degrees of freedom.

Statically Over-determined System You want to compute the stiffness matrix associated with the 3 dynamical DOFs but the system is statically over-determined and it is not a particularly easy task... On the other hand, it is not a particularly difficult task if you know what to do...

My idea for solving this problem is,

1. remove one constraint (which one? one is more convenient than the others in light of the second part of the problem),
2. add the corresponding DOF,
3. compute the flexibility matrix using the PVD for the resulting statically determined, 4–DOF system,
4. compute the stiffness matrix of the statically determined system,
5. eventually the stiffness matrix of the over-determined system is simply an appropriate partition of the stiffness matrix of the statically determined system.

As it happens, I've already computed the 4×4 flexibility matrix (using a specific ordering of the DOFs and the most logical choice for the extra DOF) and I want to share the intermediate result, corresponding to step 3 above:

$$\overline{\boldsymbol{F}} = \frac{L^3}{6EJ}\begin{bmatrix} 24 & 8 & 4 & 56 \\ 8 & 24 & 15 & 16 \\ 4 & 15 & 10 & 8 \\ 56 & 16 & 8 & 160 \end{bmatrix}.$$