Dynamics of Structures 2015-2016

January Homework

Due by Tuesday January 31st 2017

Contents

Instructions

Submit your work by email[1] not later than Tuesday, January 31st, in the form of a *typeset* PDF attachment containing your solutions. I will not accept scanned manuscripts.

For every problem you are required to describe your procedure with sufficient detail (your source code doesn't stand for a procedure description), including all the intermediate results necessary to derive the answers.

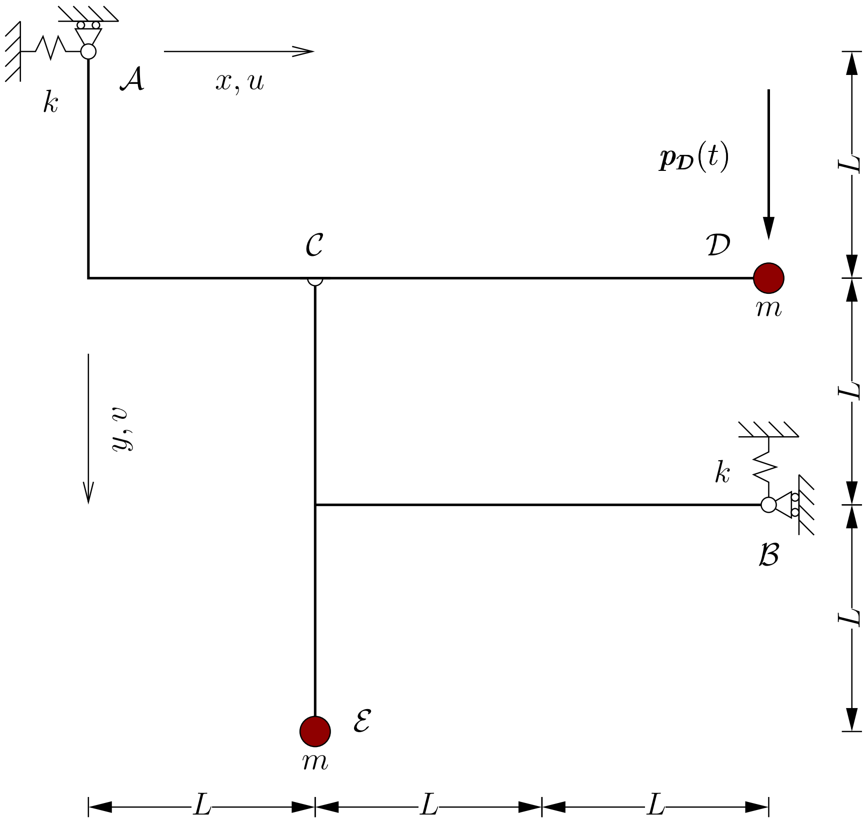
> Should you submit further, *unrequested* materials (e.g., source code, spreadsheets, endless printouts by your programs) please provide them as separate attachments. Please no `.zip` or `.rar` archives with all your files inside but instead one attachment for each file.

You can discuss the problems with your colleagues, with me and with other members of the Faculty (given that you make clear that you are discussing your homework...) but with no one else — please note that *to discuss a problem* is quite different from *sharing parts of its solution.* I will not accept homeworks with evidence of shared parts.

The maximum scores for the three problems are respectively 15 points (2 DOF), 45 (Support Motion) and 40 (Rayleigh), to be admitted to the oral exam you must score at least 60 points.

[1] Address your email to `giacomo.boffi@polimi.it`

1 2 DOF System



The system in figure is composed by two *rigid, massless* bars, supporting two equal bodies, of negligible dimensions, whose mass is m. The bars are connected by a hinge and each one is connected to the system of reference by a roller and an elastic spring, both springs having stiffness k.

1. Using $x_1 = u_{\mathcal{A}}$ and $x_2 = v_{\mathcal{B}}$ find a) the eigenvalues of the system (in terms of a reference eigenvalue $\omega_0^2 = \frac{k}{m}$) and b) the corresponding normalized eigenvectors.

The system is at rest when it is excited by a force $p_{\mathcal{D}}$,

$$p_{\mathcal{D}}(t) = k\delta \begin{cases} 0 & t \leq 0 \\ \sin 2\pi \frac{t}{T} & 0 \leq t \end{cases}$$

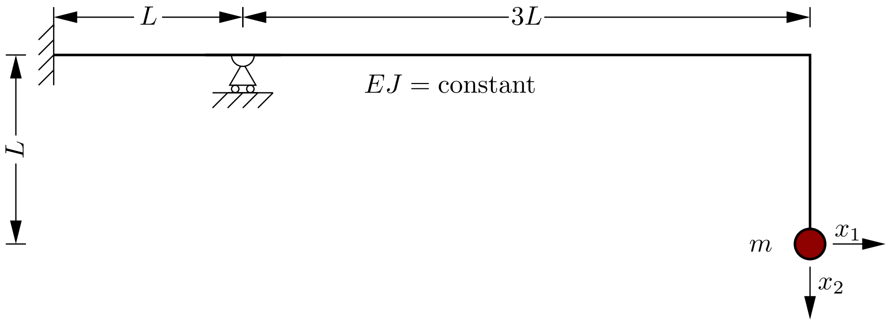
with $(\omega_0 T)^2 = 14$.

2. Give the analytical expressions of the modal responses for $0 \leq t$ (e.g., $q_1 = 7.2(\sin(2\pi t/T) - 0.75 \sin(2.66\pi t/T))\,\delta$).

3. Plot the modal responses for $0 \leq t \leq 6T$ and $30T \leq t \leq 36T$.

4. Plot the displacement $v_{\mathcal{D}}(t)$ in the same time intervals.

2 Imposed Support Motion



A structure is composed of an elastic, uniform, massless beam that supports a body of negligible dimensions and mass m.

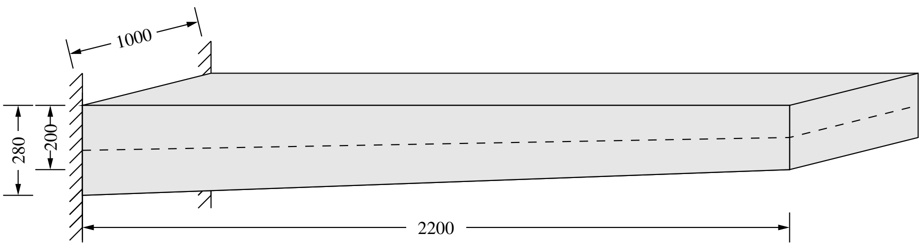
The structure is at rest when it's excited by an imposed vertical acceleration of the intermediate support (positive when downwards)

$$\ddot{v}_{\mathrm{g}} = \delta\,\omega_0^2 \begin{cases} 0 & t \leq 0 \\ \omega_0 t(3-\omega_0 t)(5-\omega_0 t) & 0 \leq \omega_0 t \leq 5 \end{cases}$$

where $\omega_0^2 = \dfrac{EJ}{ML^3}$.

Plot the dynamic deflections, the static displacements and the total displacements of the supported mass, detailing all the the intermediate steps that you had to perform, in the interval $0 \leq \omega_0 t \leq 5$.

3 Rayleigh (Ritz?) Quotient Method



A tapered cantilever, made of reinforced concrete, is $L = 2200\,\mathrm{mm}$ long, its base section is 280 mm high and 1000 mm wide, its end section is $200\,\mathrm{mm} \times 1000\,\mathrm{mm}$ and it supports at its tip a mass of 300 kg (not indicated in the figure).

With $\rho = 2500\,\mathrm{kg\,m^{-3}}$ and $E = 16\,\mathrm{GPa}$ (conventional, to account for cracking) find the best estimate of the natural period of vibration of the cantilever/mass system using the Rayleigh Quotient Method and the function

$$v(x,t) = \left(a\left(\frac{x}{L}\right)^2 + (1-a)\left(\frac{x}{L}\right)^3\right)\,Z_0\,\sin\omega t = \phi_3(x)\,Z_0\sin\omega t$$

to approximate the displacements in free vibration, where Z_0 is a dimensional scaling factor, ω is the unknown frequency of vibration and a is a free parameter.

OPTIONAL (2 extra pts) Use $\phi_4(x) = a\,\left(\frac{x}{L}\right)^2 + b\left(\frac{x}{L}\right)^3 + (1-a-b)\left(\frac{x}{L}\right)^4$ in place of ϕ_3.
HINT `Solver`