Homework due on April 6th 2017

This homework is strictly optional and has no influence at all on your final marks. Its only purpose is to let you test your preparation, it's hence important that you try to do everything on your own.

If you want our feedback please return to me, after the class of Thursday April 6th, either a printed paper or a *clearly intelligible* manuscript; your homework will be corrected and returned to you in a few days.

1 Impact

A body, its mass $m_1 = 400\,\mathrm{kg}$, is in equilibrium under a gravitational force and the reaction of an elastic support, its stiffness $k_1 = 100\,\mathrm{N\,mm^{-1}}$.

You drop a second body, its mass $m_2 = 100\,\mathrm{kg}$, on the first one from an height $h = 100\,\mathrm{mm}$.

After the impact the two bodies remain in contact or, in other words, their velocities are the same (they are *glued together*).

You measure $x_{\max}$, the maximum displacement of the masses with respect to the position of the masses at the instant of the impact (see also Figure 1) and it is $x_{\max} = 9.83\,\mathrm{mm}$.

Where are you in the Solar System?

2 Dynamic Testing

A simple one storey building, in essence a SDOF system, is tested using a vibrodyne, that applies to the building a harmonic force $p(t) = p_0 \sin \omega t$ with $p_0 = 4000\,\mathrm{N}$.

The building is tested at different frequencies ω_i, measuring the steady-state response

$$x_{\mathrm{ss},i} = \rho_i \sin(\omega_i t - \theta_i).$$

The results are summarized in Table 1 — note that the measurements, ρ_i and θ_i, were affected by errors and rounded to the nearest integer.

Give your best estimate of the characteristics of the building: mass, lateral stiffness and damping ratio.

3 Vibration Control & Numerical Integration

A machine, its mass $M = 15.2\,\mathrm{Mg}$, when it is operated starting from rest transmits to its rigid supports an unbalanced force

$$p(t) = \frac{p_0}{\omega_0^2}\left(\dot\phi^2(t)\sin\phi(t) - \ddot\phi(t)\cos\phi(t)\right) \quad \text{with}$$

$$\phi(t) = \omega_0 t_0 \begin{cases} \left(\frac{t}{t_0}\right)^2 - \frac{1}{3}\left(\frac{t}{t_0}\right)^3 & 0 \le t \le t_0 \\ \frac{t}{t_0} - \frac{1}{3} & t > t_0 \end{cases}$$

where $p_0 = 800\,\mathrm{N}$, $t_0 = 5\,\mathrm{s}$ and $\omega_0 = 2\pi \times 8\,\mathrm{rad\,s^{-1}}$ (note that for $t \ge t_0$ the unbalanced force is harmonic, $p(t) = p_0 \sin\omega_0\tau$, with constant angular velocity ω_0).

Plot the phase angle $\phi(t)$, the angular velocity $\dot\phi(t)$, the angular acceleration $\ddot\phi(t)$ and the unbalanced force $p(t)$ in the time interval $0 \le t \le 6\,\mathrm{s}$.

3.1 Vibration Control

To control ambient vibration the maximum value of the force transmitted to the rigid supports, in the steady state condition, must be limited as follows, $f_{ss} \le 200\,\mathrm{N}$.

Design two suspension systems that comply with the above requirement, for two assigned values of the damping ratio: $\zeta_1 = 0.01$ and $\zeta_2 = 0.10$.

3.2 Numerical Integration

Using the Linear Acceleration algorithm, with a time step $h = 10\,\mathrm{ms}$, determine the instantaneous force $f(t)$ that the two suspension systems transmit to the rigid support during the transient, plot the transmitted force in the interval $0 \le t \le 10\,\mathrm{s}$ and discuss your results.

4 Rayleigh Quotient

A 3 storey building is modeled as a plane 3 DOF system, neglecting the beam flexural deformations and the columns axial deformations (see also Figure 2).

The storey masses (bottom to top) are $m_1 = 3m$, $m_2 = 3m$, $m_3 = 2m$ and the interstorey stiffnesses are $k_1 = 5k$, $k_2 = 4k$, $k_3 = 2k$.

Using an initial shape vector $\boldsymbol{\phi}_0 = \begin{Bmatrix}1 & 1 & 1\end{Bmatrix}^T$ compute an approximation to the lowest frequency of vibration using the Rayleigh Quotient method and later refine your initial frequency estimate using better estimates of the maxima of the potential and kinetic energies of the freely vibrating system.

Tables

i	f_i [Hz]	ρ_i [µm]	θ_i [degrees]
1	4	177	5
2	5	218	8
3	6	300	14
4	7	517	29

Table 1: Experimental Results

Figures

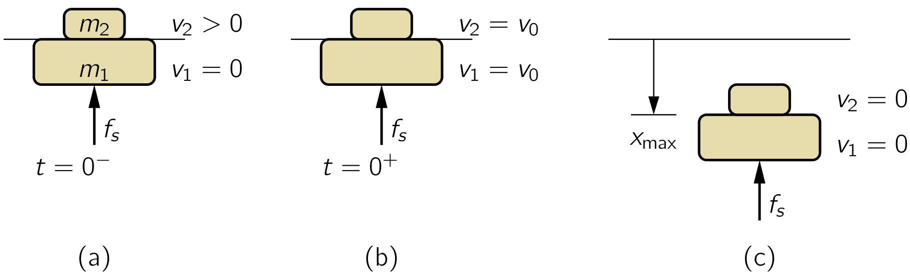

Figure 1: Impact: (a) just before the impact, (b) just after the impact and (c) maximum displacement (The masses are subjected not only to the spring force but also to inertial forces and their own weights).

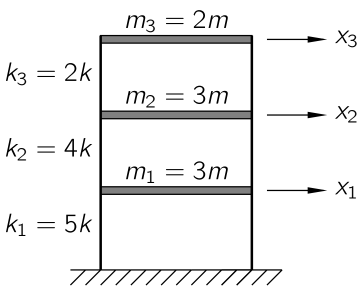

Figure 2: Building Model