Dynamics of Structures 2016–17, 3rd homework

This homework is strictly optional. I will publish the solution on Friday May 26Th, so that you can compare my solution with yours.

1 3 DOF System

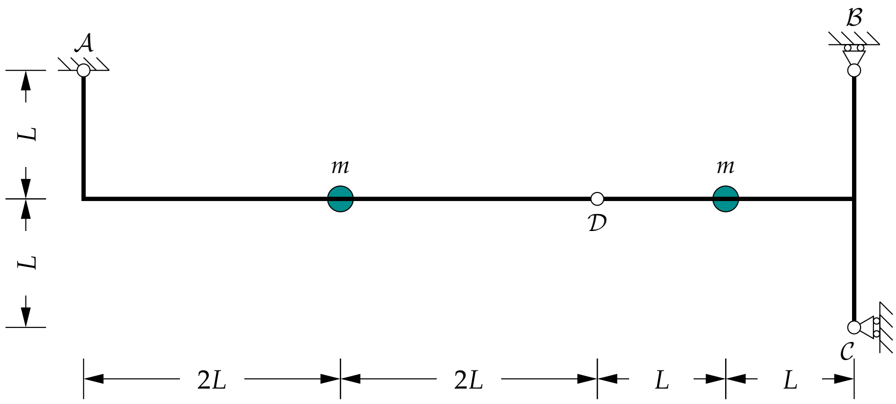

Figure 1: the 3 DOF system.

The undamped structure in figure 1 is composed of two uniform beams whose masses are negligible with respect to the masses m of the two identical bodies that the beams support; the axial and shear deformabilities of the beams are negligible with respect to their flexural deformability, characterized by the flexural stiffness EJ.

The system is at rest when it is subjected to an imposed horizontal displacement of the hinge in $\mathcal{A}$,

$$u_{\mathcal{A}} = \delta \begin{cases} 0 & t \le 0, \\ \dfrac{\bar{\omega}t - \sin(\bar{\omega}t)}{2\pi} & 0 \le t \le \dfrac{2\pi}{\bar{\omega}}, \\ 1 & \dfrac{2\pi}{\bar{\omega}} \le t \end{cases}$$

where $\bar{\omega} = {}^{4}/_{3}\,\omega_0$, with $\omega_0^2 = {}^{EJ}/_{mL^3}$.

1. Plot $u_{\mathcal{A}}$ and $\ddot{u}_{\mathcal{A}}$ (normalized with respect to δ and $\omega_0^2\,\delta$, respectively) in the time interval $0 \le t \le {}^{2\pi}/_{\omega_0}$.

2. Integrate numerically the equation of motion and plot the dynamic component of the horizontal displacement of the masses (normalized with respect to δ) in the time interval $0 \le t \le {}^{16\pi}/_{\omega_0}$.

3. Write the modal equations of motion and the analytical expressions of the modal responses, in the forced phase and in the free vibration phase.

4. Plot the dynamic component of the horizontal displacement of the masses, derived in terms of the modal responses, in the time interval $0 \le t \le {}^{16\pi}/_{\omega_0}$ and compare with the numerical solution.

Hint: for two of the 48 possible labelings of the DOFs it is $\boldsymbol{K} = \frac{3}{1588}\frac{EJ}{L^3}\begin{bmatrix} +1340 & -358 & -704 \\ -358 & +655 & -504 \\ -704 & -504 & +1280 \end{bmatrix}$.

2 Continuous System

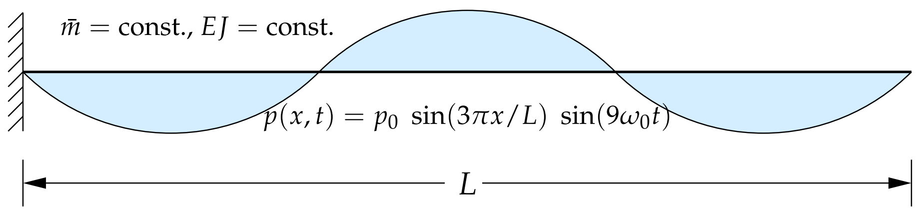

Figure 2: the continuous system.

An undamped, uniform beam of length L, unit mass $\bar{m}$ and flexural stiffness EJ is clamped at one end and it is free at the other one.

The beam is at rest when it is excited by a dynamic load,

$$p(x,t) = p_0 \begin{cases} \sin(3\pi x/L)\sin(9\omega_0 t) & \text{for } 0 \le t \le \frac{2\pi}{3\omega_0} \\ 0 & \text{otherwise.} \end{cases}$$

where $\omega_0^2 = \frac{EJ}{\bar{m}L^4}$.

1. Compute an approximation of the tip displacement, $v(L,t)$ using the first 3 modes of vibration and plot your results (normalized with respect to ${}^{p_0L^4}/_{EJ}$) in the time interval $0 \le t \le \frac{2\pi}{\omega_0}$.

2. Compute an approximation to the bending moment at the clamped end, $M(0,t)$ using the first 3 modes of vibration and plot your results (normalized with respect to p_0L^2) in the same time interval.