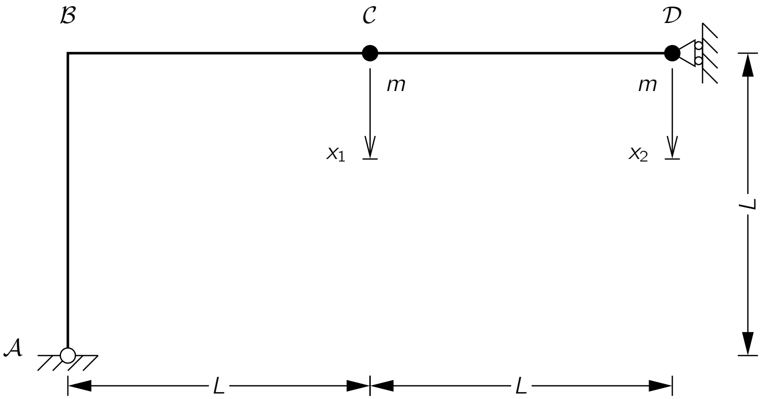
2 DOF System



The structure in figure is composed of a single uniform beam whose mass is negligible with respect to the two supported masses. Neglecting the axial deformability of the beam, the dynamic degrees of freedom are the two vertical displacements of the masses, x_1 and x_2.

The flexibility and stiffness matrices of the dynamic system, considering only the flexural deformability, are

$$\boldsymbol{F} = \frac{L^3}{6EJ}\begin{bmatrix} 4 & 9 \\ 9 & 24 \end{bmatrix}, \qquad \boldsymbol{K} = \frac{EJ}{L^3}\frac{2}{5}\begin{bmatrix} 24 & -9 \\ -9 & 4 \end{bmatrix}.$$

A vertical force is applied in $\mathcal{D}$, leading to a static vertical displacement $x_{2,0} = 1\,\text{mm}$ and at time $t = 0$ it is istantaneously removed.

Write the equation of motion $x_2 = x_2(t)$ neglecting the effects of damping.

Rayleigh Estimates

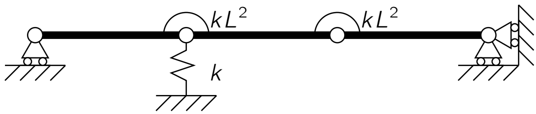
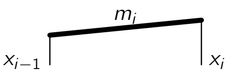


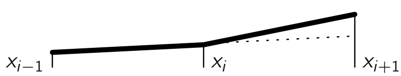
Figure 1: the rigid system.

The system in figure 1 is composed of 3 identical rigid bars, their individual length L and their individual mass m, an extensional spring and two flexural springs, their stifnesses as indicated in figure.

Using the vertical displacements of the internal hinges as the degrees of freedom and the trial vector $\boldsymbol{x}_0 = \left\{1 \quad 2\right\}^T$ give a first estimate of the fundamental frequency of vibration of the system using the Rayleigh Quotient method and a refinement of the estimate using a better approximation of the strain energy.

> *The kinetic energy of a rigid body has a contribution from the tranlational velocity of its centre of mass and another one from the rotational velocity. — The strain energy in a flexural spring depends on the* relative *rotation between the bodies it connects,* $V = \frac{1}{2}\, K(\varphi_{i+1} - \varphi_i)^2$.





$$x_{G,i} = \frac{x_i + x_{i-1}}{2}; \qquad \varphi_{i+1} = \frac{x_{i+1} - x_i}{L}, \quad \varphi_i = \frac{x_i - x_{i-1}}{L}.$$