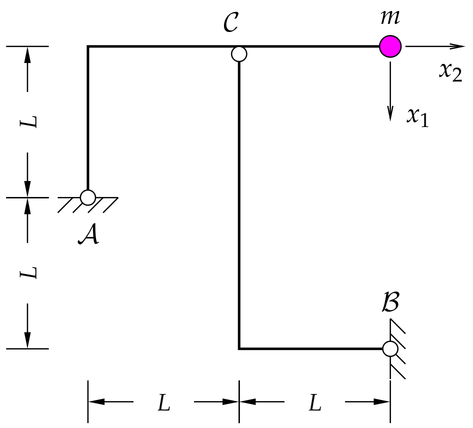
Dynamics of Structures' written test, February 15th 2018.

2 DOF System



The dynamic system in figure is composed of two uniform beams of negligible mass and a dimensionless body, its mass equal to m; the dynamic degrees of freedom are the two components of the displacement of the body, x_1 and x_2.

The flexibility and stiffness matrices of the dynamic system, considering only the flexural deformability, are

$$\boldsymbol{F} = \frac{1}{27}\frac{L^3}{EJ}\begin{bmatrix}80 & 13\\13 & 20\end{bmatrix}, \qquad \boldsymbol{K} = \frac{1}{53}\frac{EJ}{L^3}\begin{bmatrix}20 & -13\\-13 & 80\end{bmatrix}$$

where EJ is the flexural stiffness of the beams and L is indicated in figure.

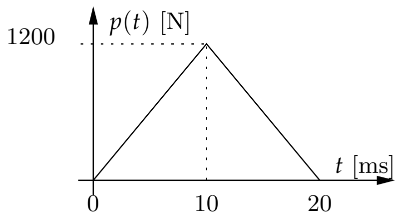
Compute the eigenvalues of the system in terms of the reference value $\omega_0^2 = EJ/(mL^3)$, i.e., $\omega_i^2 = \lambda_i^2\,\omega_0^2$ and the eigenvectors of the system, normalized with respect to the mass matrix

The system is at rest when it is subjected to a horizontal motion of the hinge in $\mathcal{B}$, $u_{\mathcal{B}}$. The modal equations of motion having been written like this

$$\ddot{q}_i + \lambda_i^2\omega_0^2\,q_i = a_i\,\ddot{u}_{\mathcal{B}}, \qquad i = 1, 2$$

determine the numerical values of the coefficients a_1 and a_2.

Single DOF Response



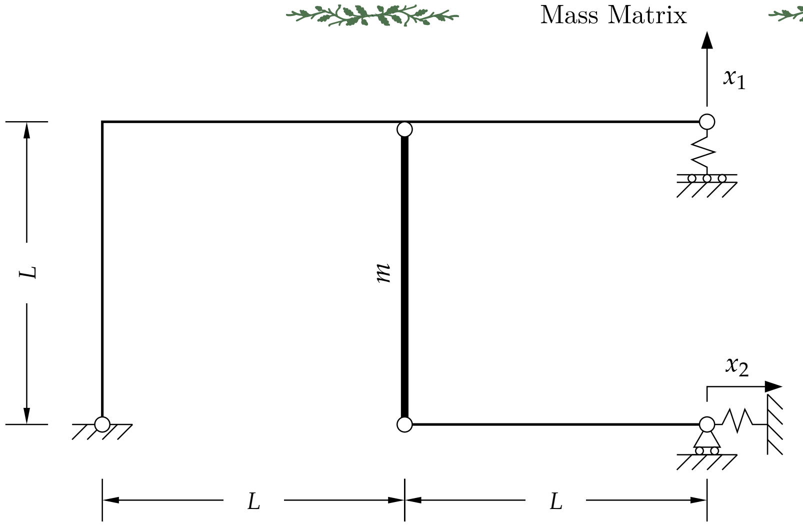
An undamped SDOF system, $m = 20\,\mathrm{kg}$, $k = 800\,\mathrm{N\,m^{-1}}$ is at rest when it is excited by an impulsive load characterized by the maximum value, $p_0 = 1.2\,\mathrm{kN}$, its duration, $\Delta t = 20\,\mathrm{ms}$ and its shape, i.e., an isosceles triangle.

a) Using the approximation to the value of the system's momentum valid for very short impulses give an estimate of the maximum displacement and of the maximum spring force.

b) Compute the system displacement and velocity at time $t_2 = 20\,\mathrm{ms}$ using the linear acceleration method with a time step $h = 10\,\mathrm{ms}$.

c) Using the results of the linear acceleration method determine the maximum displacement in the free response phase.

Mass Matrix



The dynamic system in figure is composed of three rigid bars, two massless and the central one with a total mass, uniformly distributed, equal to m.

The three bars have per se 9 degrees of freedom, the constraints (3 hinges and a roller) determine 7 of these degrees of freedom so that the system has 2 remaining DoF.

Using the degrees of freedom indicated in figure determine the mass matrix of the system in the hypothesis of small displacements.