Response of a Single DOF System

Giacomo Boffi

Here follows a straightforward implementation of the (approximate) impulse-momentum relationship, valid for durations of the loading much shorter than the natural period of the system. The integral of a triangular *curve* is of course simply the area of the triangle...

```
from math import sqrt

m = 20.0 # kg
k = 800.0 # N/m
p_max = 1200.0 # N
Delta_t = 0.02 # s
h = 0.01 # s

w = sqrt(k/m) # rad/s

Delta_moment = p_max*Delta_t/2 # N \times s = kg m / s
initial_velocity = Delta_moment/m # m / s
amplitude_of_motion = initial_velocity/w # m
```

and the result is

```
print('Variation of moment =  %.3f N s' % Delta_moment)
print('x(t_2)              =   0.000 mm')
print('v(t_2)              = %.3f mm/s' % (initial_velocity*1000))
print('Max displacement    =  %.3f mm' % (amplitude_of_motion*1000))
```

```
Variation of moment =  12.000 N s
x(t_2)              =   0.000 mm
v(t_2)              = 600.000 mm/s
Max displacement    =  94.868 mm
```

0.1 Linear Acceleration

The damping, the time step, the formula for the modified stiffness.

```
c = 0 # N / (m/s) - undamped system
h = 0.01 # s
ϰ = k + 3*c/h + 6*m/h**2
```

The initial conditions

```
x0 = 0.0
v0 = 0.0
```

The loading function and the initial acceleration

```
p0 = 0.0
p1 = 1200.0
p2 = 0.0

a0 = (p0 - k*x0 - c*v0) / m
```

We are ready for the first step, 1st the modified load, then the displacement increment, the velocity increment and the quantities at the end of the step

```
dp1 = (p1-p0) + c*(a0*h/2+3*v0) + m*(3*a0+6*v0/h)

dx1 = dp1/ϰ
dv1 = 3*dx1/h - 3*v0 - a0*h/2

x1 = x0+dx1
v1 = v0+dv1
a1 = (p1 - k*x1 - c*v1) / m
```

and the 2nd step is just the first one with all the *indices* incremented by one...

```
dp2 = (p2-p1) + c*(a1*h/2+3*v1) + m*(3*a1+6*v1/h)
dx2 = dp2/x
dv2 = 3*dx2/h - 3*v1 - a1*h/2

x2 = x1+dx2
v2 = v1+dv2
a2 = (p2 - k*x2 - c*v2) / m
```

Our results ($x_{\max}$ happens during free vibrations and its amplitude is $\sqrt{x_2^2+v_2^2/\omega_n^2}$).

```
print("x(t_0)  = %.3f mm    " % (x0*1000))
print("v(t_0)  = %.3f mm/s  " % (v0*1000))
print("a(t_0)  = %.3f m/s^2" % (a0))
print()
print("x(t_0)  = %.3f mm    " % (x1*1000))
print("v(t_0)  = %.3f mm/s  " % (v1*1000))
print("a(t_0)  = %.3f m/s^2" % (a1))
print()
print("x(t_2)  = %.3f mm    " % (x2*1000))
print("v(t_2)  = %.3f mm/s  " % (v2*1000))
print("a(t_2)  = %.3f m/s^2" % (a2))
print()
print('x_{max} = %.3f mm' % (sqrt(x2**2+v2**2/w**2)*1000))
```

```
x(t_0)  = 0.000 mm
v(t_0)  = 0.000 mm/s
a(t_0)  = 0.000 m/s^2

x(t_0)  = 0.999 mm
v(t_0)  = 299.800 mm/s
a(t_0)  = 59.960 m/s^2

x(t_2)  = 5.992 mm
v(t_2)  = 598.402 mm/s
a(t_2)  = -0.240 m/s^2

x_{max} = 94.805 mm
```