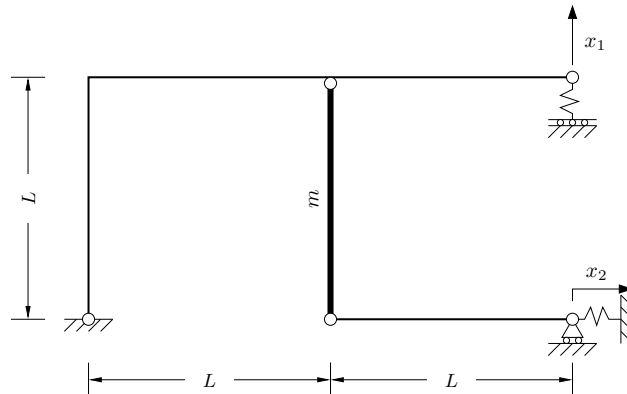


Mass Matrix of an Articulated 2 DOF System

Giacomo Boffi



The 2 DOF dynamical system in figure is composed of two massless rigid bodies and a massive one.

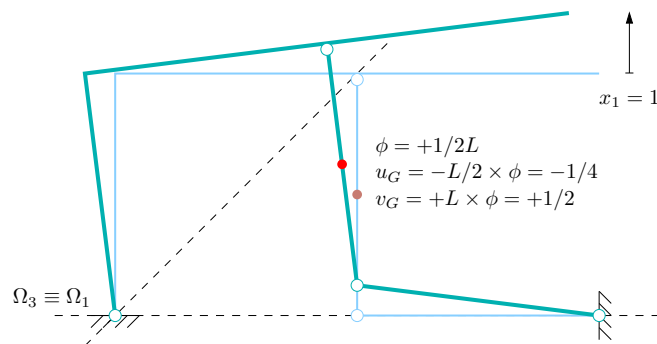
Compute the mass matrix of the system with reference to the degrees of freedom indicated in figure, in the hypothesis of small displacements.

0.1 Solution

We are going to use symbols for the relevant quantities.

```
| m, L, x1, x2 = symbols('m L x_1 x_2')
```

Contribution of x_1 to the displacements of the massive bar



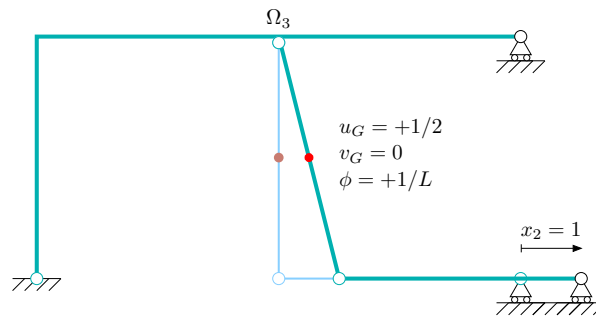
We constrain x_2 to zero (i.e., the roller becomes a hinge) and impose a unit displacement $x_1 = 1$.

The Centre of Instantaneous Rotation (CIR) of the massive bar, at the intersection of the dashed lines in figure, coincides with the CIR of the left bar, hence the rotation of the two bars are the same.

Because the two rotations are $\phi_1 = 1/2L$ the displacements of the centre of mass are $u_{G1} = -\phi_1 \times L/2 = -1/4$ and $v_{G1} = +\phi_1 \times L = 1/2$.

```
| ug1, vg1, phi1 = -x1/4, +x1/2, +x1/(2*L)
```

Contribution of x_2 to the displacements of the massive bar



We constrain x_1 to zero (i.e., we introduce a roller) and impose a unit displacement $x_2 = 1$.

The left beam can't move, hence the CIR of the massive bar is the top internal hinge. The CIR of the bottom hinge is at an infinite distance in the vertical direction (the bottom bar undergoes a horizontal motion) and by continuity we have $\phi_2 = 1/L$, $u_{G2} = -\phi_2 \times (-L/2) = +1/2$ and $v_{G2} = 0$.

```
ug2, vg2, phi2 = +x2/2, 0, +x2/L
```

Total Displacements and Velocities The total displacement components are the sum of the two contributions, the total rotation is the sum of the two contributions.

The velocities are obtained differentiating w/r to time.

```
ug, vg, phi = ug1+ug2, vg1+vg2, phi1+phi2
dot_u, dot_v, omega = diff_t(ug), diff_t(vg), diff_t(phi)
```

```
T = m * (dot_u**2 + dot_v**2 + omega**2*L**2/12) / 2
display(Latex('\$T=' + latex(T.expand()) + '\$'))
```

$$T = \frac{mx_1^2}{6} - \frac{mx_1}{12}x_2 + \frac{mx_2^2}{6}.$$

Mass Matrix Coefficients The coefficient can be computed as

$$m_{ij} = \frac{\partial^2 T}{\partial x_i \partial x_j}.$$

```
for i, xi in enumerate((x1, x2), 1):
    for j, xj in enumerate((x1, x2), 1):
        display(Latex('\$m_{%d%d}=%%(i,j)+latex(T.diff(xi,xj))+'\$'))
```

$$T = \frac{mx_1^2}{6} - \frac{mx_1}{12}x_2 + \frac{mx_2^2}{6}.$$

$$m_{11} = \frac{m}{3}.$$

$$m_{12} = -\frac{m}{12}.$$

$$m_{21} = -\frac{m}{12}.$$

$$m_{22} = \frac{m}{3}.$$

0.2 Initialization

```
from sympy import symbols, init_printing, latex
init_printing(use_latex=1)
def diff_t(expr): return expr
```