Homework due on April 5th 2018

This homework is strictly optional and has no influence at all on your final marks. Its only purpose is to let you test your preparation, it's hence important that you try to do everything on your own.

If you want our feedback please return to me, after the class of Thursday April 5th, either a printed paper or a manuscript (with intelligible handwriting); your homework will be corrected and returned to you in a few days.

1 Dynamic Testing

A simple one storey building, in essence a SDOF system, is tested using a vibrodyne, that applies to the building a harmonic force $p(t) = p_0\omega^2 \sin \omega t$.

The building is tested at different frequencies ω_i, measuring the steady-state response

$$x_{\text{ss},i} = \rho_i \sin(\omega_i t - \theta_i).$$

The results are summarized in Table 1 — note that the measurements ρ_i and θ_i were affected by errors and that the values reported in the table are rounded.

i	f_i [Hz]	p_i[N]	ρ_i [µm]	θ_i [degrees]
1	18	3240	54	24.3
2	20	4000	118	55.1
3	22	4840	132	123.9
4	24	5760	80	152.5

Table 1: Experimental Results

Give your best estimate of the characteristics of the building: mass, lateral stiffness and damping ratio.

2 Vibration Control

A machine, its mass $M = 17.13\,\text{Mg}$, when it is operated at steady-state transmits to its rigid supports an unbalanced force $f(t) = p_0 \sin \omega t$ where $p_0 = 900\,\text{N}$ and $\omega_0 = 2\pi \times 10\,\text{rad}\,\text{s}^{-1}$.

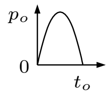
To control ambient vibration the maximum value of the force transmitted to the rigid supports, in the steady state condition, must be limited as follows, $f_{\text{ss}} \leq 300\,\text{N}$.

Design two suspension systems that comply with the above requirement for two assigned values of the suspension damping ratio, $\zeta_1 = 0.01$ and

$\zeta_2 = 0.10$ and determine the amplitude of the machine displacements at steady state.

3 Numerical Integration

A SDOF, $m = 400\,\text{kg}$, $\omega_n = 2\pi\,4\,\text{Hz}$ and $\zeta = 3\%$, is at rest when it is excited by an half-sine impulse, $t_0 = 40\,\text{ms}$ and $p_0 = 8.2\,\text{kN}$.



1. Find the maximum response, that happens during free vibrations, using the exact formula but neglecting damping.

2. Find the maximum response using the approximate formula based on the impulse-momentum relationship, neglecting damping.

3. Find the maximum response using the constant acceleration algorithm, considering damping.

4. Find the maximum response using the constant acceleration algorithm, considering that the spring is elastic-perfectly plastic with a yielding strength $f_y = 3.2\,\text{kN}$.

4 Rayleigh Quotient

A 3 storey building is modeled as a plane 3 DOF system, neglecting the beam flexural deformations and the columns axial deformations (see also Figure 1).

The storey masses (bottom to top) are $m_1 = 5m$, $m_2 = 5m$, $m_3 = 3m$ and the interstorey stiffnesses are $k_1 = 8k$, $k_2 = 5k$, $k_3 = 2k$.

Using an initial shape vector $\boldsymbol{\phi}_0 = \begin{Bmatrix}1 & 2 & 3\end{Bmatrix}^T$ compute an approximation to the lowest frequency of vibration using the Rayleigh Quotient method and later refine your initial frequency estimate using better estimates of the maxima of the potential and kinetic energies of the freely vibrating system.

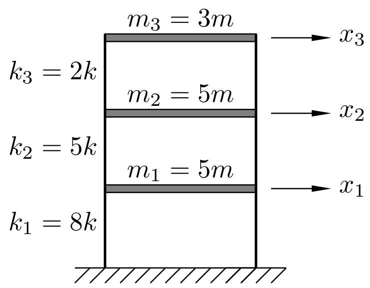


Figure 1: Building Model