Homework due on April 19th 2018

This homework is strictly optional and has no influence at all on your final marks. Its only purpose is to let you test your preparation, it's hence important that you try to do everything on your own.

If you want our feedback please return to me, after the class of Thursday April 19th, either a printed paper or a manuscript (with intelligible handwriting); your homework will be corrected and returned to you in a few days.

2 DoF System

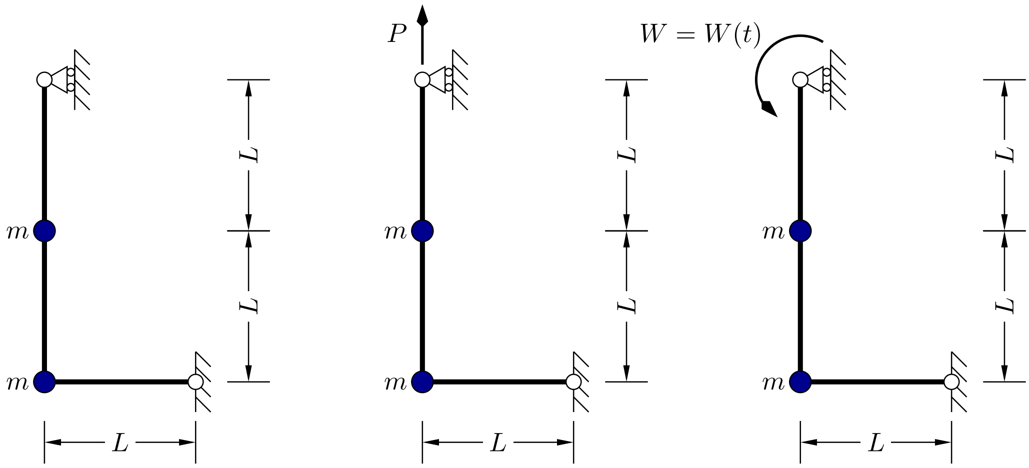

Figure 1*a*. Figure 1*b*. Figure 1*c*.

The dynamic system in figure 1*a* consists of a single, uniform beam of flexural stiffness EJ supporting two equal masses of negligible dimensions. The axial and shear deformabilities of the beam are negligible when compared to the flexural deformability, the beam mass is negligible when compared to the supported masses.

1. Determine the eigenvalues and the natural frequencies of vibration of the system in terms of a reference frequency $\omega_0 = \sqrt{EJ/mL^3}$.
2. Determine the eigenvectors of the system, normalized with respect to the structural mass matrix: $\boldsymbol{\psi}_i^T \boldsymbol{M} \boldsymbol{\psi}_i = m$.
3. With reference to figure 1*b*, the system is in static equilibrium under a constant force $P = EJ/300L^2$ when at time $t = 0$ the external force is

suddenly removed: determine the modal responses and the equation of motion of the roller, $v_r = v_r(t)$ for $t \geq 0$.

4. With reference to figure 1c, the system is at rest when it is excited by an external couple $W(t) = (EJ/400L)\sin\omega_0 t$: determine the modal responses and the horizontal motion of the central mass.

It is recommended that you plot the graphs of the modal responses and . of the requested structural components' responses in a time interval $0 \leq \omega_0 t \leq 20$

Rayleigh-Ritz

Consider a 30 storey shear type frame, with equal floor masses

$$m_i = m, \qquad i = 1, \ldots, 30$$

and variable storey stiffness

$$k_i = k \times (38 - i), \qquad i = 1, \ldots, 30$$

(note: $i = 1$ denotes the bottom storey/floor, 30 the top).

Determine the 5 lowest frequency modes of vibration (eigenvalues and eigenvectors) using the Rayleigh-Ritz method, with a base

$$\mathbf{\Phi} = [\boldsymbol{\phi}_j], \qquad j = 1, \ldots, 10$$

where

$$\phi_{nj} = \sin((j\pi - \frac{\pi}{2})\frac{n}{30}), \qquad n = 1, \ldots, 30 \text{ and } j = 1, \ldots, 10$$

(note: each $\boldsymbol{\phi}_j$ is a sine function, with a number of zeros over the height of the building equal to $j - 1$ and with the top *displacement* equal to -1^{j-1}).

In MATLAB code

```
N = 30 ; n = [1:N] ;
M = 10 ; j = [1:M] ;
Phi = sin(pi*(n/N)'*(j-1/2)) ;
```

In Python+`numpy` code

```
N, M = 30, 10
n, j = arange(N)+1, arange(M)+1
Phi = sin(pi*n[:,None]/N*(j-1/2))
```