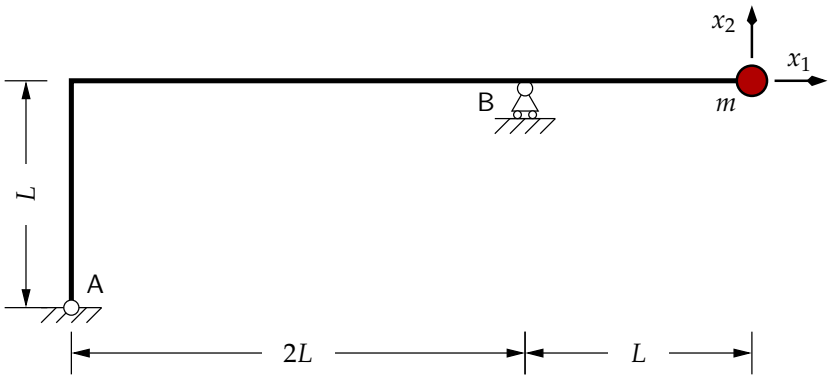
Written Test

Dynamics of Structures

July 2, 2018

2 DoF system — Support Motion

The undamped dynamic system in figure 1 is composed of a simply supported uniform beam and a supported mass. [1]



[1] The beam mass is negligible with respect to the supported mass. The system's stiffness matrix, with respect to the degrees of freedom indicated in figure 1, is

$$\boldsymbol{K} = \frac{3}{8}\frac{EJ}{L^3}\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}.$$

Figure 1: the dynamic system.

1. Determine the squared frequencies of vibration of the two modes with respect to the reference frequency $\omega_0^2 = {}^{EJ}/_{mL^3}$.
2. Determine the eigenvectors of the system.

The system is at rest when the left hinge in A is subjected to a horizontal displacement[2] $u_A = u_A(t)$,

$$u_A(t) = \Delta \begin{cases} 0 & t \le 0, \\ \dfrac{t}{t_1} - \dfrac{1}{2\pi}\sin(2\pi\dfrac{t}{t_1}) & 0 \le t \le t_1, \\ 1 & t_1 \le t. \end{cases}$$

[2] Δ is the amplitude of the final displacement and t_1 is related to the periods of vibration by the relationship $\omega_0 t_1 = \pi/8 \quad \rightarrow \quad 2\pi/t_1 = 16\omega_0$

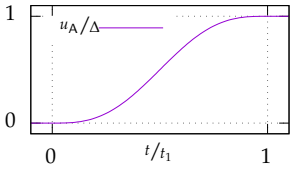


Figure 2: the displacement of the support, normalized with respect to Δ, vs time, normalized with respect to t_1.

3. Draw a sketch of the velocity and of the acceleration of the support.
4. Write the modal equations of motion for $0 \le t \le t_1$, complete of the relevant numerical values.
5. Compute $x_1({}^{t_1}/_2)$.

Rayleigh Quotient

A three storey building is modeled as a shear type frame, with the parameters indicated in figure 3.

Find your best estimate of the (squared) natural frequency of vibration using the Rayleigh Quotient method and the initial shape vector

$$\boldsymbol{\phi}_0 = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}.$$

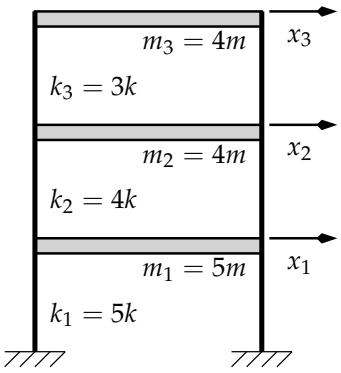


Figure 3: shear type frame.