Written Test

Dynamics of Structures

July 16, 2018

The total points for a perfect work are 36, bonus questions notwithstanding. You are required to score at least 18.

2 DoF system — Support Motion

The 2 DoF, undamped dynamic system[1] in figure 1 is composed of two uniform beams, forming a three-hinged arch, and two supported masses.

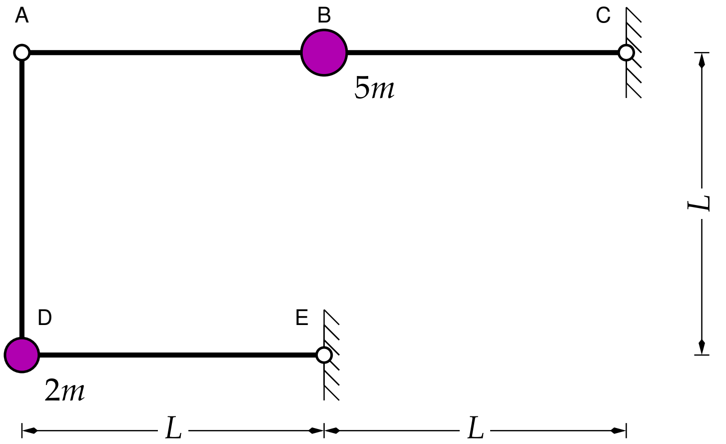


Figure 1: the dynamic system.

[1] The beams masses are negligible with respect to the supported masses. The axial and shear deformabilities are negligible with respect to the flexural deformability. The system's stiffness matrix, with respect to x_1 and x_2, respectively the downward vertical displacement of D and of B, is

$$\boldsymbol{K} = \frac{EJ}{L^3}\begin{bmatrix} 3 & -3 \\ -3 & 6 \end{bmatrix}.$$

1. Determine the squared frequencies[2] of vibration of the two modes with respect to the reference frequency $\omega_0^2 = EJ/mL^3$.
2. Determine the eigenvectors of the system.

[2] Have you noticed that the masses are different?

The system is at rest when the bottom hinge in E is subjected to a horizontal displacement to the right,

$$u_{\mathsf{E}}(t) = \Delta \begin{cases} 0 & t \le 0, \\ \dfrac{t}{t_1} & 0 \le t \le t_1, \\ 1 & t_1 \le t, \end{cases}$$

where Δ is the amplitude of the final displacement and t_1 is related to the periods of vibration by the relationship $\omega_0 t_1 = 1$.

3. Draw a sketch of the diplacement, of the velocity and of the acceleration[3] of the support.
4. Write the modal equations of motion for $0 \le t \le t_1$, complete of the appropriate numerical values.
5. Determine the initial conditions,[4] both in structural coordinates and in modal coordinates.
6. Compute $x_1(t_1)$.

[3] When the velocity is constant, the acceleration is ....

[4] In every moment, $\dot{x}_{\text{tot}} = \dot{x}_{\text{stat}} + \dot{x}$. At $t = 0^+$ you know (you should know...) both $\dot{x}_{0,\text{tot}}$ and $\dot{x}_{0,\text{stat}}$.

Bonus question: what happens for $t > t_1$?

Turn the page.

Free Vibrations

A uniform beam is connected to its system of reference by a hinge and by an elastic support (see figure 2).

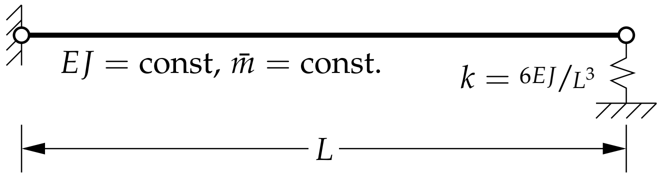


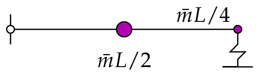
Figure 2: elastically supported uniform beam.

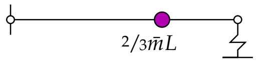
Find (an approximation to) the natural frequency of vibration using one or more[5] of the following possible methods:

[5] Every method you use, obtaining the correct result, will give you 1/6 of the total credit for the second problem.

- writing the boundary conditions for the general integral of free vibrations of a slender beam

$$\phi(x) = A\sin\beta x + B\cos\beta x + C\sinh\beta x + D\cosh\beta x;$$





- using the 2 DoF model sketched in the margin;
- using the 1 DoF model sketched in the margin;
- using the Rayleigh quotient method, with $\psi_a(x) = \sin \pi x/L$;
- using the Rayleigh quotient method, with $\psi_b(x) = x/L$;
- using the Rayleigh quotient method, [6] with $\psi(x) = a\psi_a + b\psi_b$ with $2a + b = 2$.

[6] It is $\int_0^L \psi^2 dx = \frac{La^2}{2} + \frac{2Lab}{\pi} + \frac{Lb^2}{3}$, $\int_0^L \psi''^2 dx = \frac{a^2\pi^4}{2L^3}$

Bonus question, justify the discrete models.