# Vibration Isolation

## Imports

```
from math import *
from scipy.optimize import newton
```

## Problem Data

```
M = 1200.000
f_max = 400
freq = 20.0
# max of Transmitted Force
max_tf = 150
```

### Derived data, summary

```
omega = freq*2*pi
TR = max_tf/f_max
fmt = '%30s = %f %s'
print(fmt%('Machine Mass', M, 'kg'))
print(fmt%('Unbalanced Force', f_max, 'N'))
print(fmt%('Force\'s Frequency', freq, 'Hz'))
print(fmt%('Circular Frequency', omega, 'rad/s'))
print(fmt%('Max Transmitted Force', max_tf, 'N'))
print(fmt%('Transmissibility Ratio, TR', TR, ''))
```

```
                  Machine Mass = 1200.000000 kg
              Unbalanced Force = 400.000000 N
             Force's Frequency = 20.000000 Hz
            Circular Frequency = 125.663706 rad/s
         Max Transmitted Force = 150.000000 N
    Transmissibility Ratio, TR = 0.375000
```

## Undamped Case

```
# undamped, TR = 1/(β²-1) ⇒ β² TR = 1+TR ⇒ β² = (1+TR)/TR
#           β² = ω²/(K/M) ⇒ K = ω²M/β² = ω²M TR/(1+TR)
beta2_und = (1+TR)/TR
K_und = omega*omega*M*TR/(1+TR)
print(fmt%('β² undamped', beta2_und, '(rad/s)²'))
print(fmt%('Suspension Stiffness', K_und/1E6, 'kN/mm'))
print(fmt%('Suspension Damping', 0.0, 'N/(mm/s)'))
```

```
                   β² undamped = 3.666667 (rad/s)²
          Suspension Stiffness = 5.168084 kN/mm
            Suspension Damping = 0.000000 N/(mm/s)
```

## Damped Case

```
z = 0.12
# damped, TR = sqrt(1+4ζ²β²)/sqrt((1-β²)²+4ζ²β²) ⇒
#                 ⇒ (1+4ζ²β²)/((1-β²)²+4ζ²β²) - TR² = 0
f = lambda b2: (1+4*z**2*b2)/((1-b2)**2+4*z**2*b2)-TR**2
beta2_dam = newton(f, beta2_und)
K_dam = M*omega**2/beta2_dam
dam = 2*z*sqrt(M*K_dam)
print(fmt%('β² damped', beta2_dam, '(rad/s)²'))
print(fmt%('Suspension Stiffness', K_dam/1E6, 'kN/mm'))
print(fmt%('Suspension Damping', dam/1E3, 'N/(mm/s)'))
```

```
                     β² damped = 3.913533 (rad/s)²
          Suspension Stiffness = 4.842080 kN/mm
            Suspension Damping = 18.294386 N/(mm/s)
```

## Dissipated Energy

The dissipated energy per cycle is `0` when the system is undamped, it is equal (see the margin figure) to $\pi \times x_{\max} \times f_{D,\max}$ for the damped system.

```
b2 = beta2_dam
K = K_dam

dyn_amp_fac = 1/sqrt((1-b2)**2+4*z**2*b2)
x_max = f_max/K_dam * dyn_amp_fac
v_max = x_max * omega
f_dmax = v_max * dam

print('       Max s-s displacement =', x_max*1000, 'mm')
print('           Max s-s velocity =', v_max*1000, 'mm/s')
print('      Max s-s damping force =', f_dmax, 'N')
print('Dissipated energy per cycle =', pi*x_max*f_dmax, 'J/cycle')
```

```
       Max s-s displacement = 0.02798445768194215 mm
           Max s-s velocity = 3.516630666731356 mm/s
      Max s-s damping force = 64.33460055342259 N
Dissipated energy per cycle = 0.0056560257309519486 J/cycle
```