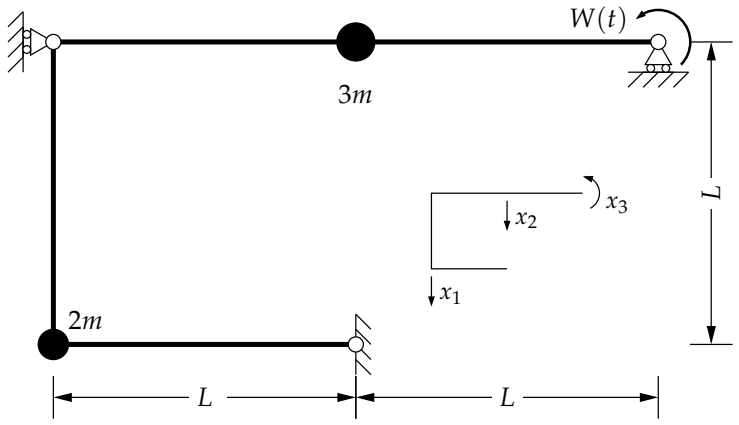
Dynamics of Structures

September 6, 2018 Written Test

A perfect score is 36. The minimum score for admission is 18.

2 DoF system — External Couple

The 2 DoF, undamped dynamic system[1] in figure 1 is composed of two uniform beams and two supported masses.



[1] The beams are slender, uniform and massless, their elastic flexural stiffness is EJ.

The system's stiffness matrix, with respect to the 3 degrees of freedom sketched in fig. 1, is

$$\bar{\boldsymbol{K}} = \frac{3}{14}\frac{EJ}{L^3}\begin{bmatrix} 15 & -20 & 4L \\ -20 & 64 & -24L \\ 4L & -24L & 16L^2 \end{bmatrix}$$

while the stiffness with respect to the dynamic degrees of freedom is

$$\boldsymbol{K} = \frac{EJ}{L^3}\begin{bmatrix} 3 & -3 \\ -3 & 6 \end{bmatrix}.$$

Figure 1: the dynamic system.

1. Determine the squared frequencies[2] of vibration of the two modes with respect to the reference frequency $\omega_0^2 = {}^{EJ}/_{mL^3}$.
2. Determine the eigenvectors of the system.

[2] Have you noticed that the two supported masses are different?

The system is at rest when a time varying couple is applied to the right top support,

$$W(t) = \Delta\frac{EJ}{L^2}\begin{cases} 0 & t \le 0, \\ \sin 2\omega_0 t & 0 \le t \end{cases}$$

where Δ is a displacement.

3. Write the modal equations of motion for $0 \le t$, complete of the appropriate numerical values.
4. Compute $x_2({}^{\pi}/_{\omega_0})$.

Vibration Isolation

At steady-state an industrial washing machine (mass $M = 1200\,\mathrm{kg}$) transmits to its rigid support a harmonic force of 400 N at 20 Hz.

Design two suspension systems, with different damping ratios $\zeta_1 = 0$ and $\zeta_2 = 12\%$, to reduce the transmitted force to 150 N.

How much energy/cycle is dissipated at steady state by the two suspension systems? Could you sketch the graph of the instantaneous dissipated power *vs* time (i.e., Watts *vs* seconds)?

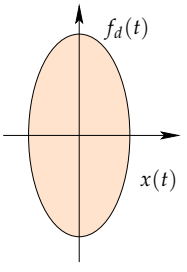


Figure 2: viscous force *vs* displacement at steady-state

Turn the page.

Continuous System

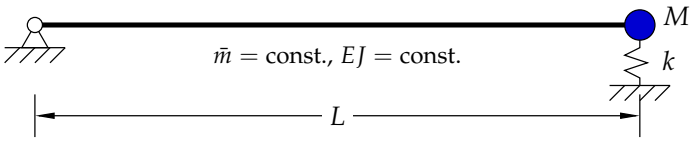


Figure 3: continuous system

The straight, uniform beam in figure 3 is simply supported at the left and it is supported by an elastic spring[3] at the right. Moreover it supports a lumped mass[4] at the right end.

Determine the boundary conditions and the first wavenumber/natural frequency of the system.

Check your results using the Rayleigh quotient method.

[3] $k = 24\,EJ/L^3$

[4] $M = 8\,\bar{m}L$