RayleighQuotient

Giacomo Boffi

Rayleigh Quotient

We use as free coordinates the rotations of the two bars wrt the horizontal direction, the strain energy is proportional to the spring stiffness and to the squared relative rotation, the kinetic energy can be written in terms of the kinetic energy of the centres of mass *PLUS* the kinetic energy associated with the rotation velocity of the two bars.

$$V = \frac{1}{2}\left(2K\theta_1^2 + K(\theta_2 - \theta_1)^2\right),$$
$$T = \frac{1}{2}\left(m\dot{x}_1^2 + m\dot{x}_2^2 + J\dot{\theta}_1^2 + J\dot{\theta}_2^2\right)$$

where

$$x_1 = \theta_1\frac{L}{2},$$
$$x_2 = \theta_1 L + \theta_2\frac{L}{2}.$$

We write the displacements (and implicitly the velocities) in terms of an assigned shape vector (here $\{12\}^T$), a reference generalized displacement Θ_0 and a harmonic function:

$$\boldsymbol{\theta}(t) = \begin{Bmatrix}1\\2\end{Bmatrix}\Theta_0 \sin\omega t.$$

Substituting these values in the expressions above we can compute the max values of the strain and kinetic energy and eventually the approximate values of the smallest eigenvector and the natural period of vibration.

```
from fractions import Fraction
from math import pi, sqrt

K, M, L, w = 1, 1, 1, 1 # dummy values...
J = Fraction(M*L**2,12)

t1, t2 = 1, 2 # trial vector: theta_1 = 1, theta_2 = 2

x1 = Fraction(t1*L, 2) # displacement of the centre of mass of 1
x2 = Fraction((2*t1+t2)*L, 2) # ditto for 2
```

```python
V = Fraction(2*K*t1**2+K*(t2-t1)**2, 2)
T = Fraction(M*((w*x1)**2+(w*x2)**2)+J*((w*t1)**2+(w*t2)**2), 2)

O2 = V/T

To = 2*pi/sqrt(O2)

print('theta_1 =', t1, 'Theta_0')
print('theta_2 =', t2, 'Theta_0')
print()
print('V  = 1/2 ', 2*V, 'K Theta_0^2')
print('T  = 1/2 ', 2*T, 'M Theta_0^2 omega^2')
print()
print('omega^2 =', O2, 'omega_0^2')
print('omega_0 T_n =', '%.3f'%To)
```

```
theta_1 = 1 Theta_0
theta_2 = 2 Theta_0

V  = 1/2  3 K Theta_0^2
T  = 1/2  14/3 M Theta_0^2 omega^2

omega^2 = 9/14 omega_0^2
omega_0 T_n = 7.837
```