Dynamics of Structures

January 29, 2019 Written Test

A perfect score is 36. The minimum score for admission is 18.

2 DoF System

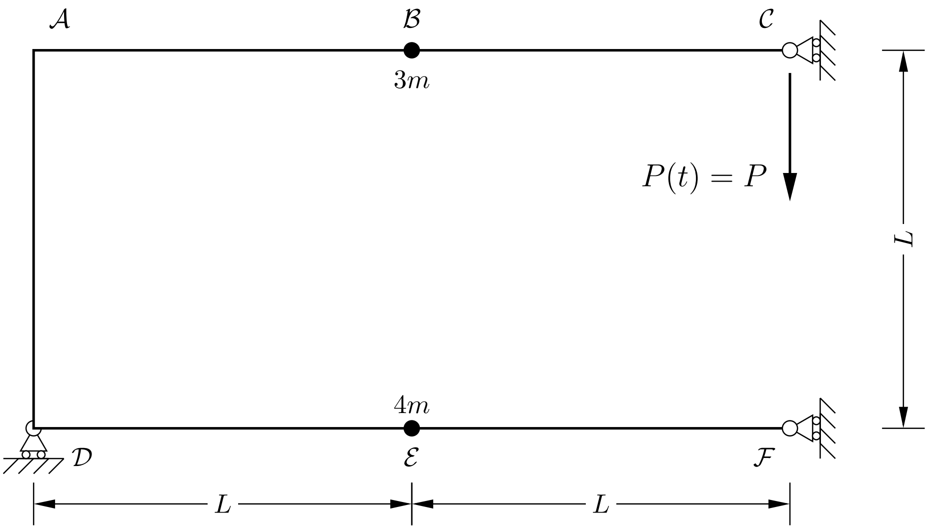

Figure 1

The undamped dynamic system in figure 1 is composed of a single, uniform beam, its flexural stiffness $EJ = \text{const.}$, supporting two *different* masses.

Neglecting the beam mass and axial deformability, we have two dynamic DoF x_1 and x_2, here the vertical displacement of $\mathcal{B}$ and $\mathcal{E}$ respectively.

Eigenvalues and Eigenvectors

1. Determine the eigenvalues of the system in terms of a reference value $\omega_0^2 = {}^{EJ}/_{mL^3}$, with the provision that the system's stiffness matrix is

$$\boldsymbol{K} = \frac{2}{5}\frac{EJ}{L^3}\begin{bmatrix}4 & 1\\1 & 4\end{bmatrix}.$$

2. Determine the eigenvectors of the system.

Please consider to set $\psi_{11} = \psi_{22} = 1$ and to normalize not the eigenvectors.

Dynamic Response

The system is undeformed and at rest when it is excited by a *constant* vertical force P, suddenly applied at $t = 0$ at point $\mathcal{C}$

$$P(t) = \begin{cases} 0 & t < 0, \\ P & t \geq 0. \end{cases}$$

> As you know, to study the dynamic response one has to introduce an additional degree of freedom x_3, the vertical displacement of $\mathcal{C}$. Taking into account the additional DoF the augmented stiffness matrix $\overline{\boldsymbol{K}}$ of the 3 DoF system is
>
> $$\overline{\boldsymbol{K}} = \frac{3}{28}\frac{EJ}{L^3}\begin{bmatrix} 92 & 6 & -34 \\ 6 & 15 & -1 \\ -34 & -1 & 15 \end{bmatrix}.$$

Providing the numerical values of all the coefficients involved,

3. write the equation of motion in nodal coordinates for the 2 DoF system,
4. write the equation of motion in modal coordinates and
5. write the expressions of the modal responses for $t \geq 0$.

Numerical Integration

A dynamic system, its mass $m = 1315\,\mathrm{kg}$, its stiffness $k = 325\,\mathrm{kN\,m^{-1}}$ and its viscous damping $c = 15.7\,\mathrm{kN\,s\,m^{-1}}$ is at rest when it is subjected to a dynamic loading

$$p(t) = 1.155\,\mathrm{kN} \begin{cases} sin(\pi t/t_0) & 0 \leq t \leq t_0 \\ 0 & \text{otherwise,} \end{cases}$$

where $t_0 = 60\,\mathrm{ms}$.

Find the displacement and the velocity of the system at $t = t_0$ using the constant acceleration algorithm and a time step $h = 20\,\mathrm{ms}$.

Rayleigh Quotient

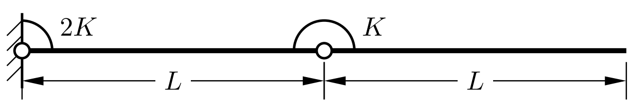

Figure 2

The system in figure 2 is composed of two uniform rigid bars, each one having a mass $m = \overline{m}L$, connected one to the other and to the system of reference by two *different* flexural springs.

Using the Rayleigh quotient method find a first approximation to the natural period of vibration of the system.

> *Hint:* θ_1, θ_2.