Homework

To have your homework corrected, please hand it on Thursday, March 21st.

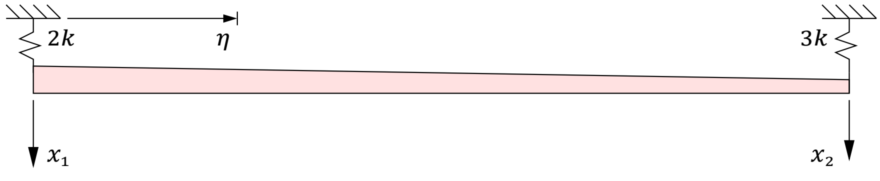
1 Initial Conditions

A single degree of freedom system has mass $m = 42$ kg, stiffness $k = 32\ \mathrm{kN\,m^{-1}}$ and damping ratio $\zeta = 2.7\,\%$.

At $t = 0$ the system is not in equilibrium, with $x_0 = 200$ mm and $\dot{x}_0 = -800\ \mathrm{mm\,s^{-1}}$, and is loaded by a dynamic load $P(t) = P_0\, exp(-at)$ with $P_0 = 700$ N and $a = 1.3\ \mathrm{s^{-1}}$.

1. Write the expression of the system response, $x(t)$ for $t \geq 0$.
2. Write the first time derivative of the response, $\dot{x}(t)$.
3. Plot the deflections of the system in the time interval $0 \leq t < 2$ s as well as the static deflections, i.e., $P(t)/k$.

2 Rayleigh Quotient



The system in figure is composed of a rigid bar and two springs. The rigid bar is non uniform, its unit mass $\mu(\eta) = (2 - \eta)\mu_0$, where $0 \leq \eta \leq 1$ is a non-dimensional coordinate.

1. Verify that the total mass of the bar is $^3/_2\mu_0 L$.
2. Using the free coordinates shown in figure (or $x \equiv x_1$, $\delta = x_2 - x_1$) compute an approximate value of the smallest eigenvalue of the system.

3 Structural Testing

A structure that can be analyzed as a SDOF system was subjected to a static load, $P = 48\,\text{kN}$, the resulting static displacement being $x_0 = 12\,\text{mm}$. The load was suddenly removed and the successive maxima of the free vibrations response were recorded: $x_1 = 11.41\,\text{mm}$, $x_2 = 10.85\,\text{mm}$, $x_3 = 10.32\,\text{mm}$ and $x_4 = 9.81\,\text{mm}$, but due to an error the times of occurrence of the maxima were not recorded.

Which structural parameters can you derive from these results? What are their values?

4 Vibration Isolation

A machine, its mass $M = 18\,000\,\text{kg}$, when it is operated starting from rest transmits to its rigid supports an unbalanced force

$$p(t) = \frac{p_0}{\omega_0^2}\left(\dot{\phi}^2(t)\,sin\,\phi(t) - \ddot{\phi}(t)\,cos\,\phi(t)\right)$$

where ϕ is the phase angle describing the unbalanced mass position, with

$$\phi(t) = \omega_0 t_0 \begin{cases} \left(\frac{t}{t_0}\right)^2 - \frac{1}{3}\left(\frac{t}{t_0}\right)^3 & 0 \le t \le t_0 \\ \frac{t}{t_0} - \frac{1}{3} & t > t_0 \end{cases}$$

where $p_0 = 1000\,\text{N}$, $t_0 = 6\,\text{s}$ and $\omega_0 = 2\pi \times 10\,\text{rad}\,\text{s}^{-1}$.

1. Plot the phase angle $\phi(t)$, the angular velocity $\dot{\phi}(t)$, the angular acceleration $\ddot{\phi}(t)$ and the unbalanced force $p(t)$ in the time interval $0 \le t \le 8\,\text{s}$.

2. Design two suspension systems, with the two assigned values of the damping ratio: $\zeta_1 = 0.01$ and $\zeta_2 = 0.12$, so that the transmitted force at steady-state is $f_{\text{s-s}} \le 300\,\text{N}$.

3. Using the Linear Acceleration algorithm, with a time step $h = 8\,\text{ms}$, determine the peak values of the instantaneous force $f(t)$ transmitted to the support during the transient, plot the transmitted force in the interval $0 \le t \le 10\,\text{s}$ and discuss your results.