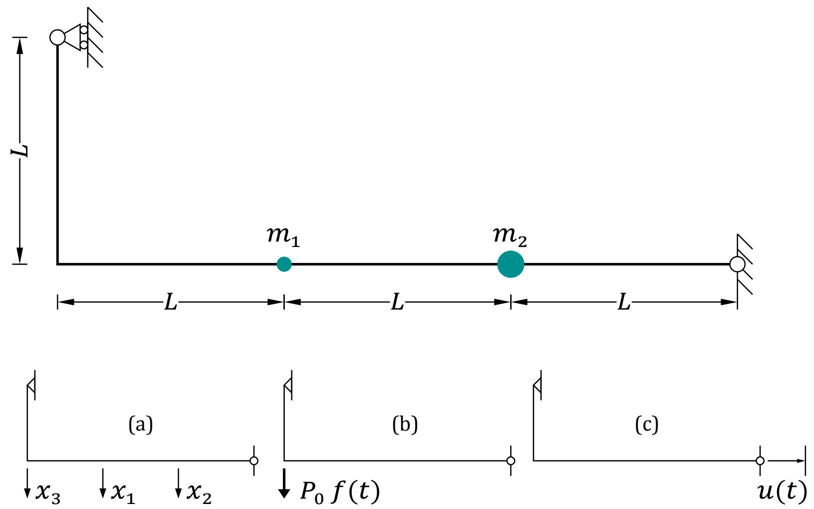
Homework no. 2

To have your homework corrected, please hand it on Thursday, April 4^{ts}.



A single,uniform, simply supported beam supports two masses $m_1 = m$ and $m_2 = 3m$. Neglecting the axial deformability and the beam's mass, the system has two dynamic degrees of freedom, x_1 and x_2 as indicated in sub-figure (a). For the degrees of freedom x_1, x_2 and x_3 (again, see sub-fig. (a)) the flexibility matrix is

$$\mathbf{F} = \frac{1}{6}\frac{EJ}{L^3}\begin{bmatrix} 48 & 27 & 58 \\ 27 & 16 & 32 \\ 58 & 32 & 72 \end{bmatrix}$$

1. For the dynamic 2 DoF system determine: (a) the mass matrix $\mathbf{M}$, (b) the stiffness matrix $\mathbf{K}$, (c) the eigenvalues, normalized with respect to $\omega_0^2 = {}^{EJ}/_{mL^3}$, (d) the eigenvectors, normalized using $\psi_{1i} = 1$ for $\forall i$, (e) the modal mass matrix $\mathbf{M}^*$ and (f) the modal stiffness matrix $\mathbf{K}^*$.

With reference to sub-figure (b), the system is at rest when it is affected by a dynamic load acting on the non-dynamic DoF x_3; with $\omega_0 t_0 = 4\pi$ it is

$$p(t) = \begin{Bmatrix} 0 \\ 0 \\ P_0 \end{Bmatrix} f(t), \qquad \text{with } f(t) = \begin{cases} 4\frac{t}{t_0}\left(1 - \frac{t}{t_0}\right) & 0 \le t \le t_0, \\ 0 & \text{otherwise.} \end{cases}$$

2. Find the equivalent load vector, acting on the dynamic degrees of freedom, using the static condensation procedure.

3. Write the two equations of motion in modal coordinates.

4. Determine the particular integrals and write the expressions of modal responses for $0 \le t \le t_0$.

5. Plot the modal responses in the same interval.

6. Write the expressions of the masses' displacements in the same interval.

7. Plot the masses' displacements.

8. Compare with the results of numerical integration — use the constant *and* the linear acceleration algorithm with the same time step $h = T_1/12$, where $T_1 = 2\pi/\omega_1$ is the natural period of vibration.

With reference to sub-figure (c), the system is at rest when it is affected by an imposed horizontal displacement of the hinge at the bottom right,

$$u(t) = \delta \begin{cases} 0 & t \le t_0, \\ \frac{t}{t_0} - \frac{sin(2\pi\, t/t_0)}{2\pi} & 0 \le t \le t_0, \\ 1 & t_0 \le t, \end{cases}$$

where δ is the final value of the imposed displacement (note that the horizontal displacement of the roller is equal to zero).

9. Plot $u(t)$ and $\ddot{u}(t)$ in the interval $0 \le t \le t_0$.

10. Sketch the rigid motion of the beam that is possible when you downgrade the hinge to a roller to permit its horizontal displacement.

11. Determine the influence matrix **E**.

12. Write the equations of motion in modal coordinates.

13. Write the expressions of the particular integrals and of the modal responses.

14. Plot the *total* displacement of the left mass.