Earthquake Excitation

Giacomo Boffi

http://intranet.dica.polimi.it/people/boffi-giacomo

Dipartimento di Ingegneria Civile Ambientale e Territoriale
Politecnico di Milano

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Outline

Seismic Excitation

The most important quantity related to earthquake excitation is the ground acceleration.
Ground acceleration can be recorded with an accelerometer, basically a SDOF oscillator, with a damping ratio $\zeta \approx 70\%$, whose displacements are proportional to ground accelerations up to a given frequency.
Instrument records of *strong ground motion* first became available in the ’30s, the first record of a destructive ground motion being the 1940 records of El Centro earthquake.

Seismic Excitation

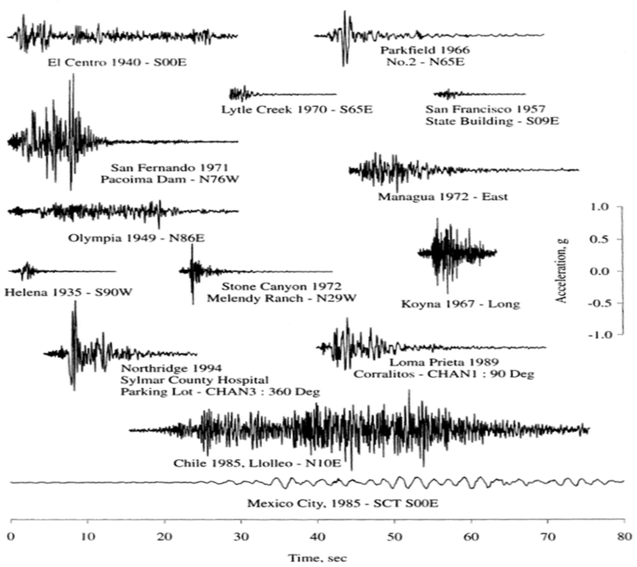
Historically, most of the strong motion records were recorded for a few earthquakes, in California and Japan, in different places and different locations (in the free field, on building foundations, on different building storeys etc), while a lesser number of records were available for different areas.

In more recent years, many national research agencies installed and operated networks of strong motion accelerometers, so that the availability of strong motion records, recorded in different geographic areas and under different local conditions is constantly improving.

Moreover, in many countries the building codes require that important constructions must be equipped with accelerometers, further increasing the number of available records.

`https://ngawest2.berkeley.edu/`,
`http://itaca.mi.ingv.it/ItacaNet_30/`.

Seismic Excitation Samples



A number of different strong motion records, recorded at different sites and due to different earthquakes, are plotted with the same scale, both in time and in acceleration.

Appreciate the large variability in terms of amplitudes, duration and frequency content of the different records.

We need a method to categorize this variability.

Detailed Sample

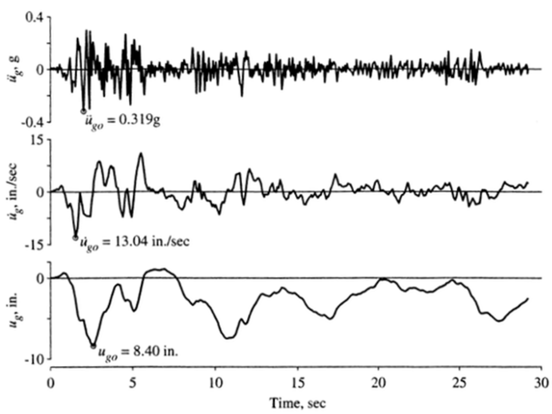


Figure 6.1.4 North–south component of horizontal ground acceleration recorded at the Imperial Valley Irrigation District substation, El Centro, California, during the Imperial Valley earthquake of May 18, 1940. The ground velocity and ground displacement were computed by integrating the ground acceleration.

On the left, the acceleration recorded at El Centro during the Imperial Valley 1940 EQ, along with the velocity and displacements obtained by numerical integration.

For meaningful results, the initial conditions of integration and the removal of linear trends from the acceleration record are of capital importance (read: don't try this at home).

Detailed Sample

Strong motion being a very irregular motion, a high sampling frequency is required to accurately describe it.
Modern digital instruments record the acceleration at a rate of 200 and more samples per second and minimize the need for sophisticated correction of the accelerations before the time integration.

About Ground Motion

In the following, we consider the so called *free-field* records. A free-field record is recorded on ground free surface in a position that is deemed free from effects induced by building response.
Tough accelerations vary with time in a very irregular manner, the variation is fully known, and for an individual record we can write the equation of motion in terms of the displacement response function $D(t)$,

$$\ddot{D} + 2\zeta\omega\dot{D} + \omega^2 D = -\ddot{u}_g(t).$$

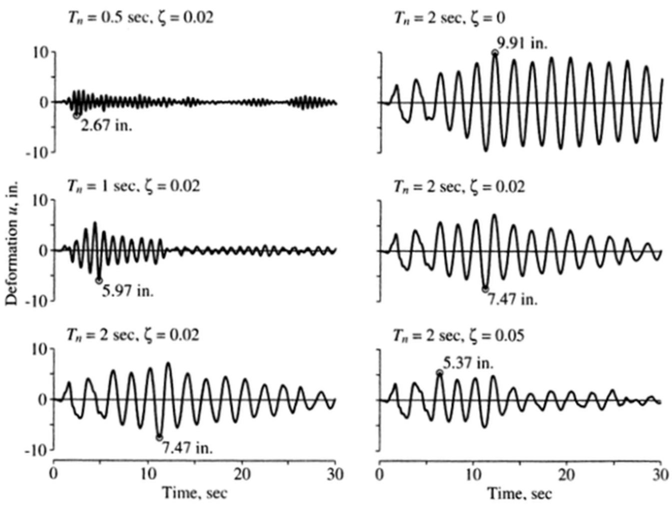
Clearly, the displacement response function, for assigned $\ddot{u}_g$, depends on ζ and ω only.

Of course, due to the irregular nature of ground excitation the response must be evaluated numerically.

Our first step will be to explore the dependency of D on ω (or rather T_n as it is usual in earthquake engineering) and ζ.

T_n and ζ dependency

Leftmost column, fixed $\zeta = 0.02$ and $T_N = 0.5, 1.0, 2.0$ s. Although the ground motion is irregular, the responses have a similarity, each one having a period close to T_n.



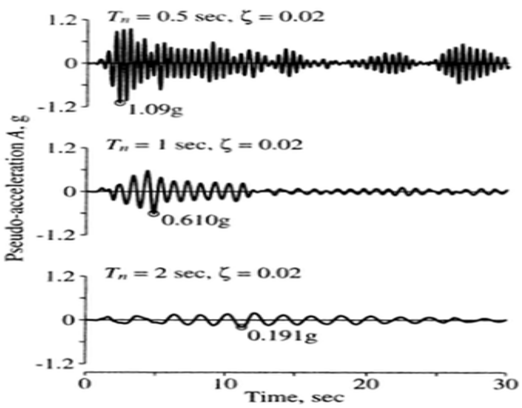
Centre column, fixed $T_n = 2.0$ s and $\zeta = 0, 0.02, 0.05$. For a fixed period the shapes are similar while the maximum response values depends on ζ.

Pseudo Acceleration

From deformation response, we can compute the equivalent static force

$$f_s(t) = m\omega^2 D(t) = mA(t)$$

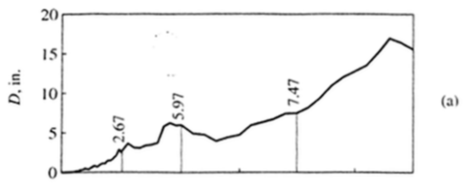
where $A(t)$ is the pseudo acceleration, $A(t) = \omega_n^2 D(t) = (2\pi)^2 D(t)/T_n^2$.
Again, note that f_s is proportional to $A(t)$ and not to the acceleration $\ddot{D}(t)$.



Left, pseudo accelerations computed for varying T_n. Compare with previous page's figure. The relative magnitudes are reverted: for $T_n = 0.5$ s we have a maximum force and a minimum displacement, while for $T_n = 2.0$ s the force is minimum and the displacement maximum.

Response Spectrum

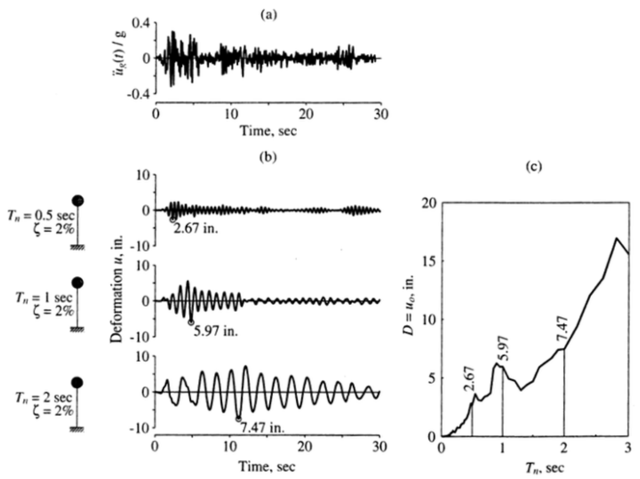
Introduced by M.A. Biot in 1932, popularized by G.W. Housner, the concept of response spectrum is fundamental to characterize e.q. response.



The response spectrum is a plot of the peak values of a response quantity, say the displacement response function, computed for different values of T_n and the same ζ, versus natural period T_N.

A graph where several such plots, obtained for different values of ζ, representative of different damping ratios that characterize different structures, are plotted close to each other represents the e.q. characteristics from the point of view of peak structural response (wait later slides for examples).

Computing the DRS



For a fixed values of ζ (usually one of 0% 0.5% 1% 2% 3% 5% 7% 10% 15% and 20%) and for variable values of T_n (usually ranging from 0.01 s to 20 s)

1. the displacement response function is numerically integrated,
2. the peak value is individuated,
3. the peak value is plotted.

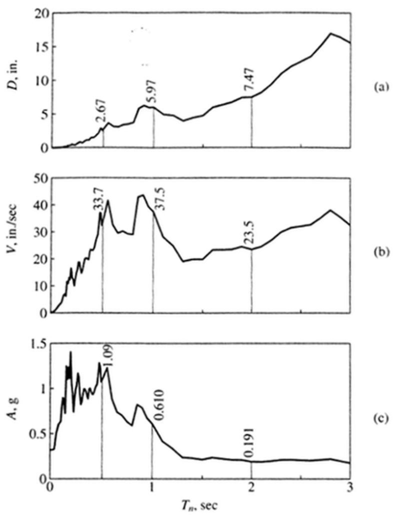
Pseudo Spectra

Only the Deformation Response Spectrum (DRS) is required to fully characterize the peaks of deformations and equivalent static forces.
It is however useful to study also the pseudo acceleration (PARS) and pseudo velocity (PVRS) spectra, as they are useful in understanding excitation intrinsic characteristics, in constructing *design spectra* and to connect dynamics and building codes.
We have already introduced $A(t)$, consider now the quantity

$$V(t) = \omega_n D(t) = \frac{2\pi}{T_n} D(t)$$

that is, the pseudo velocity.
The peak value of V is connected with the maximum strain energy, $E_{s,0} = {}^1\!/_2 m V_0^2$ being $E_{s,0} = {}^1\!/_2\, D_0\, f_{s,0} = {}^1\!/_2\, D_0\, m\omega^2 D_0$.
Once again, $V \neq \dot{x}$, the relative velocity.

Diagrams of the 3 Pseudo Spectra



Deformation spectrum,
pseudo velocity and
pseudo acceleration spectra
for El Centro 1940 NS, $\zeta = 2\%$.

Combined $D - V - A$ spectrum

In the following, we will use the symbols D, V and A to represent the values of the DRS, PVRS and PARS spectra, respectively, with

$$V = \omega_n D, \qquad A = \omega_n^2 D$$

While D, V and A represent the same information, nonetheless it is useful to maintain a distinction as they are connected to different response quantities, the maximum deformation, the maximum strain energy and the maximum equivalent static force.
Moreover, it is possible to plot all three spectra on the same logarithmic plot, giving what is regarded as a fundamental insight into the ground motion characteristics.

Constant A

Consider a plane with axes $\log T_n$ and $\log V$, and the locus of this plane where A is constant, $A = \hat{A}$:

it is

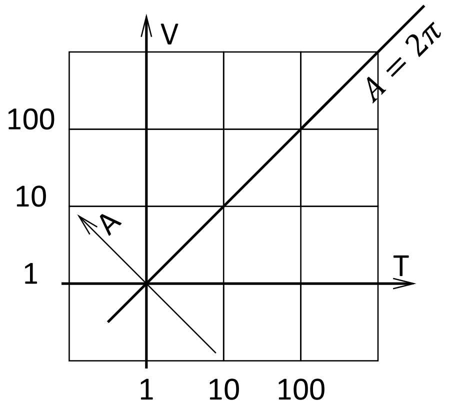
$$A = 2\pi V / T_n = \hat{A}$$

taking the logarithm

$$\log \frac{\hat{A}}{2\pi} = \log V - \log T_n$$

or

$$\log V = \log T_n + \log \frac{\hat{A}}{2\pi}.$$



In the log-log plane straight lines at 45° are characterized by a constant value of A.

Constant D

In the same plane with axes $\log T_n$ and $\log V$ we seek the locus where D is constant, $D = \hat{D}$:

it is

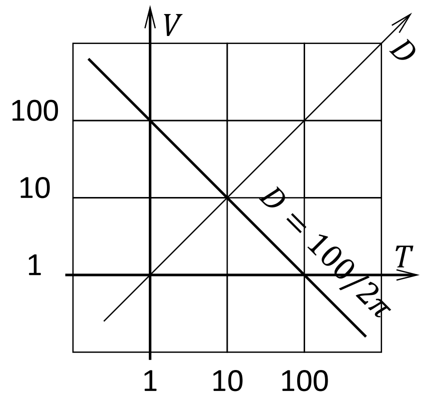
$$D = T_n V / 2\pi = \hat{D}$$

taking the logarithm

$$\log 2\pi \hat{D} = \log V + \log T_n$$

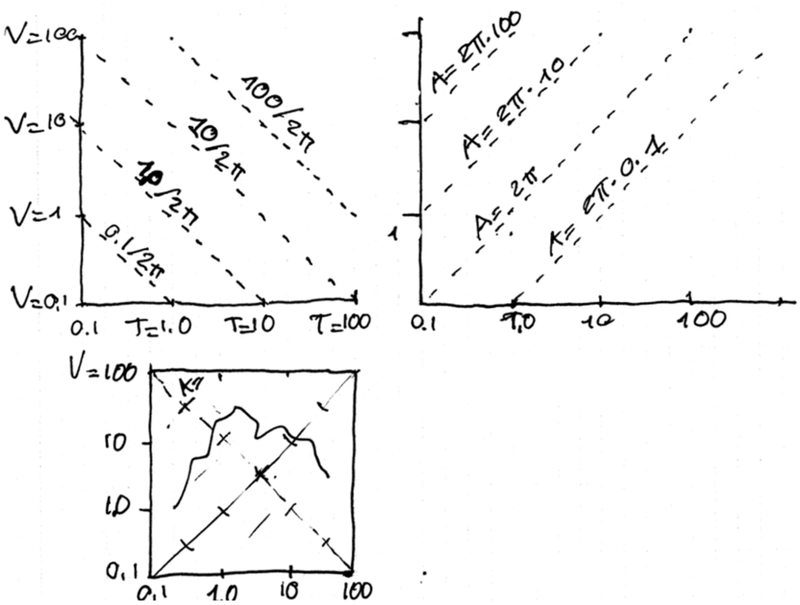
or

$$\log V = \log 2\pi \hat{D} - \log T_n.$$

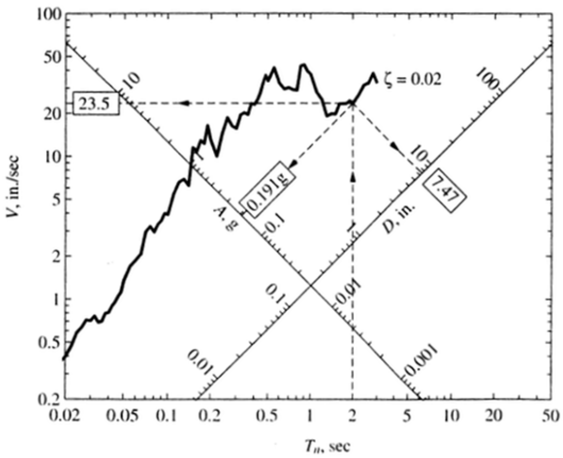


In the log-log plane straight lines at −45° are characterized by a constant value of D.

Example of Construction, 1

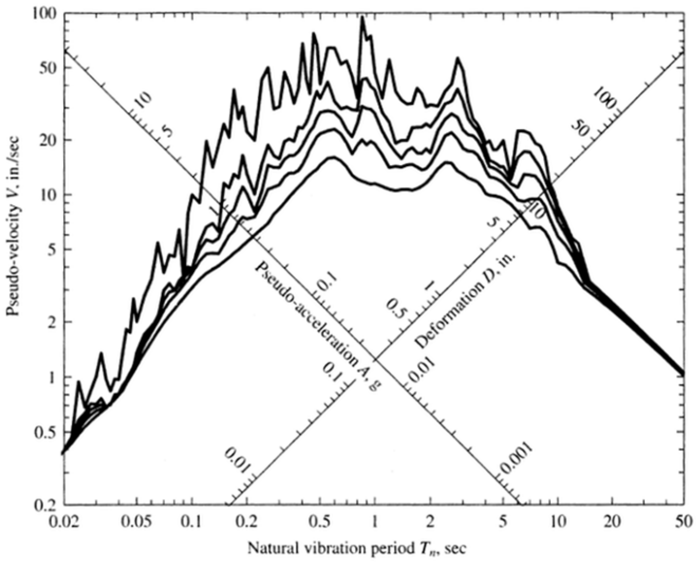


Example of Construction, 2



Combined $D - V - A$ response spectrum, El Centro 1940, NS record, $\zeta = 0.02$.

Example of $D - V - A$ spectrum



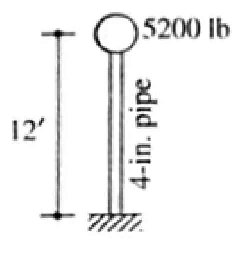
Combined $D - V - A$ response spectrum, El Centro 1940 NS record, for $0 \leq \zeta \leq 20\%$ and full range of periods.

Peak Structural Response

The peak deformation u_0 is given by $u_0 = D$ and the peak of the equivalent static force $f_{S,0}$ is given by $f_{S,0} = ku_0 = m\omega^2 u_0 = kD = mA$

Example

It is required to know the peak of the base bending moment for the structure on the right, when subjected to the NS component of the El Centro 1940 record.



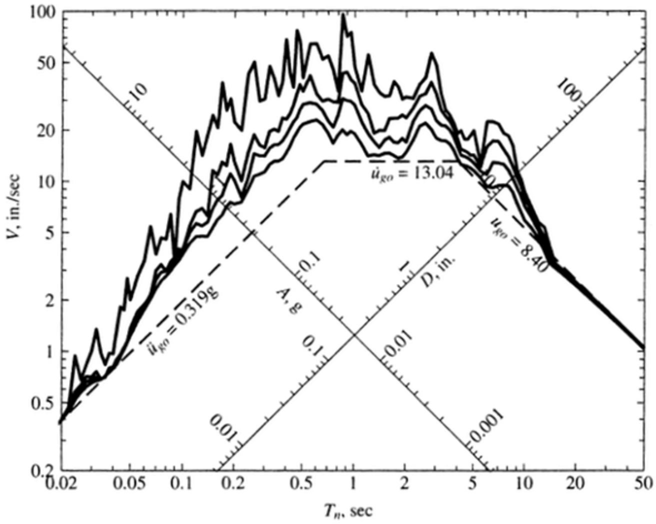
(a)

The mass is $m = 2360$ kg, the stiffness is $k = 36.84$ kN m^{-1}, the natural period of vibration is computed as $T_n = 1.59$ s. The damping ratio is assumed to be $\zeta = 5\%$.

On the graph of the relevant $D - V - A$ spectrum, for $T_n = 1.59$, we find the value $A = 0.20$ g.

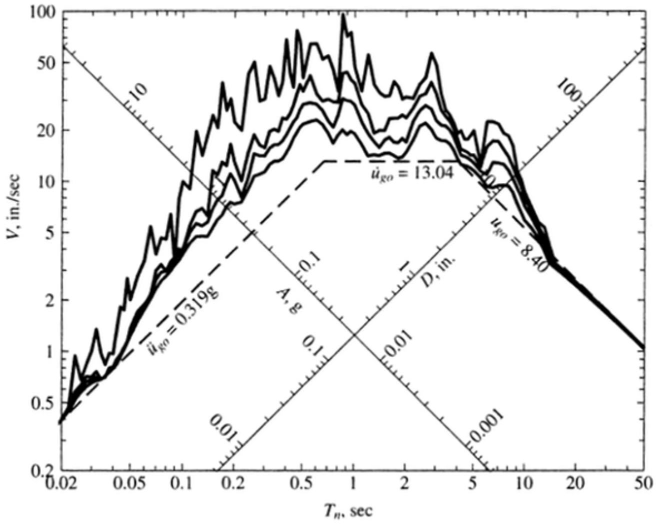
The equivalent static force is $f_{S,0} = 2360\,\text{kg} \cdot 0.20 \cdot 9.81\,\text{m s}^{-2} = 4.63\,\text{kN}$ and the peak base bending moment is $M_{b,0} = 4.63\,\text{kN} \cdot 12 \cdot 0.305\,\text{m} = 16.93\,\text{kN m}$

Response Spectrum Characteristics



The El Centro 1940 NS $D - V - A$ spectrum, *plus* three lines corresponding to the peak values of the ground acceleration, ground velocity and ground displacement.

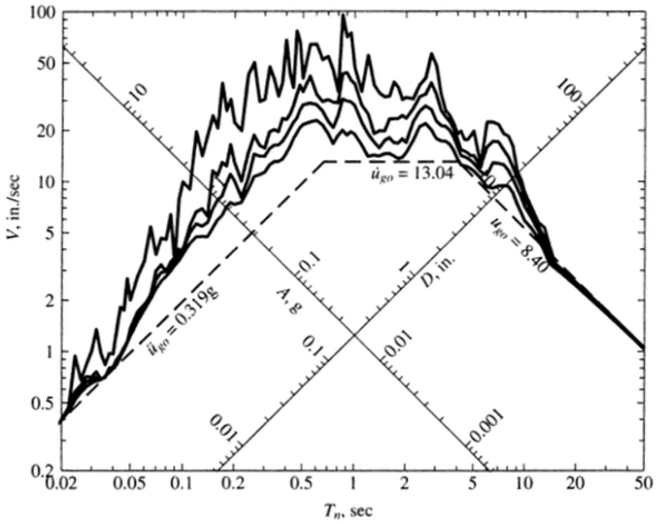
Response Spectrum Characteristics



For each value of the damping ratio, from theoretical considerations

$\lim A = \ddot{u}$ $\quad$ $\lim D = u$

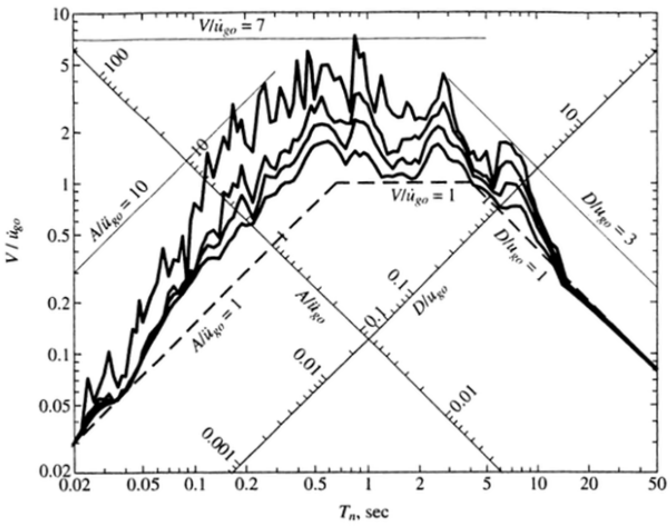
Response Spectrum Characteristics



For intermediate values of T_n it is apparent that

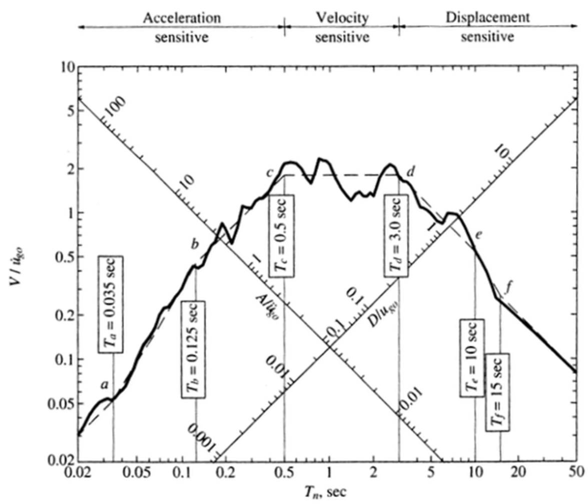
- $A > \ddot{u}_{g0}$, $V > \dot{u}_{g0}$ and $D > u_{g0}$;

Idealized Response Spectrum, 1



First step in the construction of an idealized $D - V - A$ response spectrum is to make a tripartite plot with all three ordinate axes normalized with respect to $u_{g,0}$, $\dot{u}_{g,0}$ and $\ddot{u}_{g,0}$.

Idealized Response Spectrum, 1



Next,

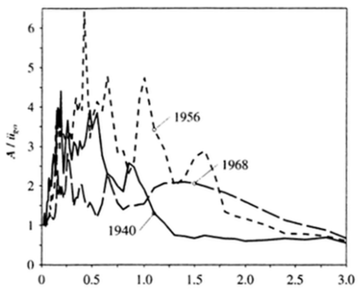
- draw the $\zeta = 5\%$ spectrum,
- individuate the intervals where
  a) $A \approx \ddot{u}_{g,0}$,
  b) $A \approx \alpha_A \ddot{u}_{g,0}$, c) $V \approx \alpha_V \dot{u}_{g,0}$,
  d) $D \approx \alpha_D u_{g,0}$, e) $D \approx u_{g,0}$
  and
- individuate approximate amplifications factors, α_A, α_V and α_D,
- connect the constant value intervals with straight lines.

Idealized Response Spectrum

Our procedure results look good in the log-log graph, but should we represent the same piece-wise linearization in a lin-lin graph it will be apparent that's a rather crude approximation.
This consideration is however not particularly important, because we are not going to use the idealized spectrum in itself, but as a guide to help developing design spectra.
Finally, consider that the positions of the points $T_a, \dots, T_f$ and the amplifications factors α_A, α_V and α_D are not equal for spectra of different earthquakes recorded at different sites, they depend in complex and not fully determined ways on different parameters, for example the focal distance and the focal mechanism and, very important, the local soil characteristics, showing in the whole a large variability.

Elastic Design Spectra

On the right, the $\ddot{u}_{g,0}$ normalized A response spectra for 3 different earthquakes NS records, recorded at the same El Centro site.
Clearly, it is not possible to infer the jagged appearance of the 1968 spectra from the 1940's and 1956's ones.



For design purposes, however, it is not necessary to know in advance and in detail the next quake's response spectra as it suffices to know some sort of an upper bound on spectral ordinates, that is a *Design Spectrum*.

Elastic Design Spectra

A design spectrum is usually specified as an idealized response spectrum, as a set of connected straight lines on the log-log $D - V - A$ plot, and has not, in contrast with a response spectrum, a jagged appearance.

Note that straight lines on a log-log graph map on straight or curved lines on conventional $T - n - A$ plots.

The requirements of a design spectrum are manifold, but mostly important a design spectrum must be an envelope of possible peak values.

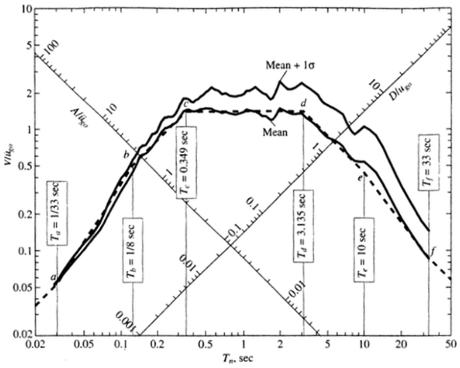
Elastic Design Spectra

The procedure used for computing an elastic design spectrum could be sketched as follows,

- collect earthquake records from the site under study or from similar sites (similar in local geology, in epicentral distances, duration of strong motion etc) and compute normalized response spectra,
- statistically characterize, in terms of mean values and standard deviations, the set of normalized spectral ordinates at hand,
- derive idealized spectra.

Derivation of an Elastic Design Spectra

Riddel and Newmark (1979)



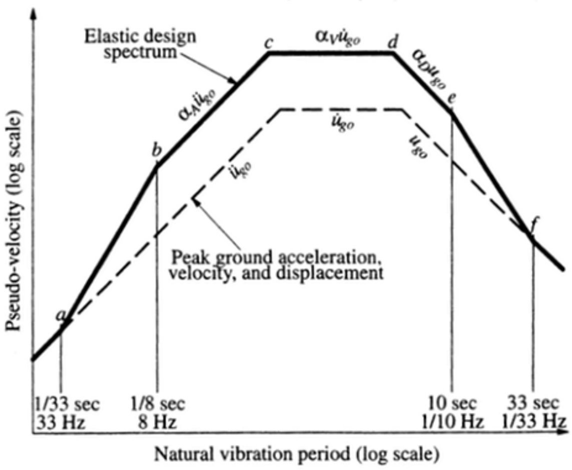
Riddel and Newmark

- a collected a large set of records for similar sites in Southern California,
- b computed the normalized response spectra for $z = 5\%$ and, finally
- c computed the mean value and the standard deviation of the peak response distribution.

In the graph, the summary of their research: the mean and mean+1σ spectra for 5% damping ratio.
In the same graph, you can see also (dashed) an idealized spectrum representation of the mean spectrum.

Idealized Elastic Design Spectra

It is common practice to subdivide the design $D - V - A$ elastic spectrum in 7 segments and use 4 key vibration periods, together with given amplification factors, to draw the required idealized design spectrum.



The key periods $T_a = 0.03$ s and $T_b = 0.125$ s define the segment where A rises from 1 to α_A
The key periods $T_e = 10$ s and $T_f = 33$ s define the segment where D decreases from α_D to 1.
The key periods T_c and T_d, instead, follows from applying the given amplification factors to pseudo accelerations, pseudo velocities and deformation.

Example Data

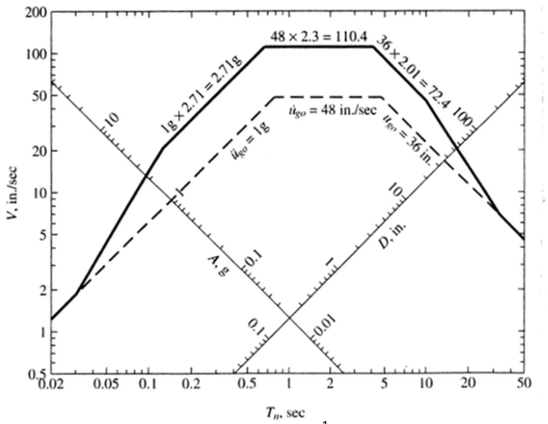
	Median (50th percentile)			Median+1σ (84th percentile)		
ζ (%)	α_A	α_V	α_D	α_A	α_V	α_D
1	3.21	2.31	1.82	4.38	3.38	2.73
2	2.74	2.03	1.63	3.66	2.92	2.42
5	2.12	1.65	1.39	2.71	2.30	2.01
10	1.64	1.37	1.20	1.99	1.84	1.69
20	1.17	1.08	1.01	1.26	1.37	1.38

	Median	Median+1σ
α_A	$3.21 - 0.68 \log \zeta$	$4.38 - 1.04 \log \zeta$
α_V	$2.31 - 0.41 \log \zeta$	$3.38 - 0.67 \log \zeta$
α_D	$1.82 - 0.27 \log \zeta$	$2.73 - 0.45 \log \zeta$

Source: N.M. Newmark and W.J. Hall, *Earthquake Spectra and Design*, EERC Report 1982.

Procedure Summary

1. For the site in case, get an estimate of $\ddot{u}_{g,0}$, $\dot{u}_{g,0}$ and $u_{g,0}$, from an analysis of relevant data or deriving it from literature,
2. in the tripartite graph, draw a line for each of the shaking parameters,
3. for a selected value of ζ amplify the shaking parameters by appropriate amplification factor and draw a line for each amplified parameter,
4. draw vertical lines from the key periods to individuate the connection ramps,
5. draw the idealized design spectrum.

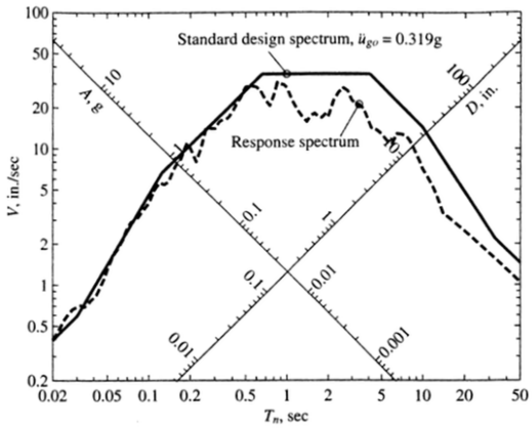


A common assumption, used when only the $\ddot{u}_{g,0}$ estimate is available, is $\dot{u}_{g,0} = \ddot{u}_{g,0}\frac{120\,\text{cm s}^{-1}}{g}$ and $u_{g,0} = \ddot{u}_{g,0}\frac{90\,\text{cm}}{g}$.

Comparison of Design and Response Spectra, 1

In the figure, the response spectrum for the 1940 El Centro NS acceleration record, computed for $\zeta = 5\%$, and the corresponding design spectrum, with amplifications corresponding to median values of the ordinates.

The spectrum was constructed from the real value of $\ddot{u}_{g,0} = 0.319\,g$ and estimated values of $\dot{u}_{g,0} = \ddot{u}_{g,0}\frac{48\,\text{inch/s}}{g} = 15.3\,\text{inch/s}$ and $u_{g,0} = \ddot{u}_{g,0}\frac{90\,\text{cm}}{g} = 11.5\,\text{inch}$, estimated values that are significantly higher than the effective values.

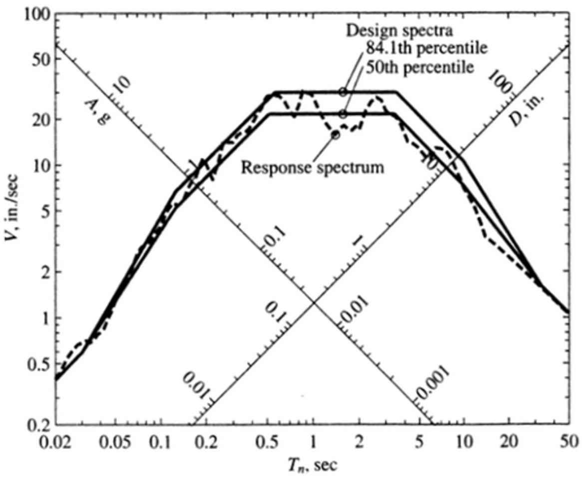


There is a good concordance in the acceleration controlled part of the design spectrum, but spectral velocities and deformations are not very good, due to rather poor estimates of the relevant ground motion peak quantities.

Comparison of Design and Response Spectra, 2

In this second slide the design spectra are two, the median and the median + 1σ versions, both based on *exact* peak values of the ground motion.

While the median spectrum is, OK, in a median position with respect to the ordinates of the elastic response spectrum, the presumed envelope spectrum does effectively a good job, maxing out most of the spikes present in the elastic response spectrum.



Differences between Response and Design Spectra

The response spectrum is a description, in terms of its peak effects, of a particular ground motion.

The design spectrum is a specification, valid for a site or a class of sites, of design seismic forces.

If a site falls in two different classifications, e.g., the site is near to a seismic fault associated with low magnitude earthquakes and it is distant from a fault associated with high magnitude earthquakes, with the understanding that the frequency contents of the two classes of events are quite dissimilar the design spectrum should be derived from the superposition of the two design spectra.

Differences between Response and Design Spectra